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A Corrected Model for the Radial Acceleration of a Rotating Object

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Abstract

The textbooks of engineering mechanics well described the term for the radial acceleration of the rotating objects with variable velocity about a fixed point. The known expressions for the radial acceleration do not have exact mathematical processing that yields vague results. Engineering computes the inertial forces as the product of the mass and acceleration that is a subject of an exact solution. The value of the inertial force reflects on the reliability and quality of the mechanism work. Analysis of analytical approaches for the modeling of the radial acceleration shows the mathematical processing can have different solutions. Mathematics is an exact science that should not give the duality in results. With several solutions, mathematical logic should base the final decision. This work considers correct mathematical processing for the radial accelerations of a rotating object about a fixed point that is the subject of mathematical physics.

Keywords: radial acceleration; rotating object; fixed point; centrifugal force; tangential velocity

Introduction

The acceleration analysis of the rotating object about the fixed point is fundamental in the textbooks of classical mechanics. The rotating object with variable angular velocity generates radial and tangential inertial forces. The textbooks of classical mechanics contain the chapters of the acceleration of the rotating objects that presented the vagueness in their analytical approach (Taylor, 2004; Gregory, 2006; Hibbeler, 2010; Dreizler, 2010; Jewett, 2018; Smith, 2018; Sparapany, 2016). The expression of the radial acceleration of the rotating objects got by the wrong formulation of the initial conditions of the angular velocity and acceleration (Nolting, 2016; Aardema, 2005; Thornton, 2004; Baumann, 2003). The radial acceleration of the object rotating with the variable angular velocity (Eq. (1)) presented in the textbooks is not acceptable for the exact sciences and yields inaccurate results in engineering (Millard, 2006; Kevin, 2019).

$$\alpha_r = r(\omega_{in} + \varepsilon t)^2 \quad (1)$$

where α_r is the radial acceleration; r is the radius of rotation of the object about a fixed point; ε is the angular acceleration; ω_{in} is the initial angular velocity; t is the time.

The inertial force acting in mechanisms is computed on a base of their mass and acceleration. The wrong value of the inertial force results in the worsening of the technical parameters and the quality of the machine's work. It presented the correct expression of the acceleration of the rotating object with the variable angular velocity in the publication (Usubamatov, 2018).

$$a_r = r\left[\omega_{in}^2 + \frac{3}{2}\varepsilon t\omega_{in} + (\varepsilon t)^2\right] \quad (2)$$

where all parameters are as specified above.

The value of the radial acceleration computed by Eq. (2) is considerably less than by Eq. (1). Consequently, also the value of the inertial force acting on the mechanism is considerably less. This important result in the analytical solution for the radial acceleration of the rotating object with the variable angular velocity. The expression of the radial acceleration (Eq. (2)) obtained by the one analytical approach but mathematics has several methods that can be applied for solving the same problem. This work presents another solution for the radial acceleration of the rotating object with the variable angular velocity, which expression is different from Eq. (2).

Methodology

The analytical approach for the solution of the radial acceleration for a rotating object with the variable angular velocity about a fixed point is the same as presented in the publication [14]. Figure 1 shows two positions of the object p that are separated by an angle α. The values of linear velocities of the object p at two positions are different. The velocity polygon of the rotating object *p* is vectorially solved for these changes in the velocity, $V_c = V_{fn} - V_{in}$. The vector, V_c is presented by two vectors that are radial velocity V_r and tangential one V_t, $V_c = V_r + V_t$. The tangential velocity of any point on an object rotating about a fixed point is determined as the product of angular velocity ω and a radius *r* of rotation, $V_t = r\omega$. The radial velocity vector V_r is directed toward the center *o* of rotation and crosses the vector velocity V_{fn} at the point *f*.

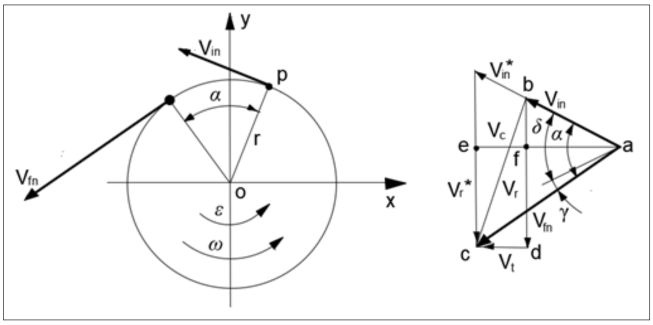

Figure 1: Velocity vectors polygon of the rotating object.

The variable linear velocity of an object *p* is given by the equation $V_{fn} = V_{in} + \varepsilon rt$,where $V_{in} = r\omega_{in}$, ω_{in} - an initial angular velocity of an object, and εrt is an extra velocity of a moving point due to an acceleration ε one, and t is a time. The radial velocity vector V_r presents the sum of two vectors $V_{r.bf}$ and $V_{r.fd}$, of segments *bf* and $fd = ec$, respectively, $V_r = V_{r.bf} + V_{r.fd}$ (Fig. 1). The two components are expressed by the following equations, $V_{r.bf} = V_{in} \sin \delta/2$ and $V_{r.fd} = V_{fn} \sin(\delta/2 + \gamma)$. The small angle α is the sector of Fig. 1 that has the expression

$$\alpha = \omega_{in} t + \frac{\varepsilon t^2}{2} \text{ where } \alpha = \delta + \gamma, \delta = \omega_{in} t \ \gamma = \alpha - \delta = \frac{\varepsilon t^2}{2}$$

Defined parameters are substituted into the expression V_r that yields:

$$V_r = V_{in} \sin \delta/2 + V_{fn} \sin(\delta/2 + \gamma) \quad (3)$$

For the small angel sinδ = Δδ. Then, the change of the radial velocity V_r is:

$$\Delta V_r = V_{in}(\Delta\delta/2) + V_{fn}(\Delta\delta/2 + \Delta\gamma) \quad (4)$$

where defined above expressions of $V_{in} = \omega_{in}$, $V_{fn} = \omega_{in} r + \varepsilon rt$, $\Delta\delta = \omega_{in}\Delta t$,and $\Delta\gamma = \varepsilon\Delta t^2/2$, which the change in the angle depends on the change in the time. Substituting defined parameters into Eq. (4),transformation and simplification yield:

$$\Delta V_r = \omega_{in}^2 r\Delta t + \frac{\omega_{in} r\varepsilon\Delta t^2}{2} + \frac{\varepsilon t\ \omega_{in}\Delta t}{2} + \frac{\varepsilon^2 t\ \Delta t^2}{2} \quad (5)$$

Equation (5) is presented by the following attitude:

$$\frac{\Delta V_r}{\Delta t} = a_r = \omega_{in}^2 r\frac{\Delta t}{\Delta t} + \frac{\omega_{in} r\varepsilon\Delta t^2}{2\Delta t} + \frac{\varepsilon tr\omega_{in}\Delta t}{2\Delta t} + \frac{\varepsilon^2 tr\Delta t^2}{2\Delta t}$$

$$= \omega_{in}^2 r + \frac{\omega_{in} r\varepsilon\Delta t}{2} + \frac{\varepsilon tr\omega_{in}}{2} + \frac{\varepsilon^2 tr\Delta t}{2} \quad (6)$$

According to the mathematical rules, the limit of Eq. (6) yields the expression of the radial acceleration of a rotating object with the variable angular velocity.

$$a_r = \lim_{\Delta t \to 0}\left(\omega_{in}^2 r + \frac{\omega_{in} r\varepsilon\Delta t}{2} + \frac{\varepsilon tr\omega_{in}}{2} + \frac{\varepsilon^2 tr\Delta t}{2}\right) =$$

$$r\left(\omega_{in}^2 + \frac{\varepsilon t\omega_{in}}{2}\right) \quad (7)$$

where all parameters are as specified above

Analysis of Eqs. (7) and (2) demonstrates the considerable difference. Equation (7) gives fewer values of the radial accelerations than Eq. (2).

Case Study

The rotating object is running with the initial angular velocity of 10 rad/s, accelerates at 7 rad/s2, and has a radius of rotation of 0.1 m. Determine the value of the radial acceleration for the object after 8 seconds of rotation.

Solution.

The radial acceleration by Eq. (7) is:

$$a_r = r\left(\omega_{in}^2 + \frac{\varepsilon t\omega_{in}}{2}\right) = 0{,}1 \times \left(10^2 + \frac{7 \times 8 \times 10}{2}\right) = 38{,}0 \ \ m/s^2 \quad (8)$$

The radial acceleration by Eq. (2) is:

$$a_r = r\left[\omega_{in}^2 + \frac{3}{2}\varepsilon\omega_{in} + (\varepsilon t)^2\right] = 0{,}1\left[10^2 + \frac{3}{2} \times 7 \times 8 \times 10 + (7 \times 8)^2\right] = 407{,}6 \text{ m/s}^2 \quad (9)$$

The result for the radial accelerations of a rotating object by new Eq. (7) yields many times less value than by Eq. (2). Such difference in the results is important for taking a solution in engineering.

Results and Discussion

The result got for the radial acceleration of a rotating object with the variable angular velocity about a fixed point by new Eq. (7) gives the many times less value than by Eq. (2) of known publications. The analysis of the steps of the analytical approaches for solutions of the radial accelerations (Eqs. (2) and (7)) shows their differences. The mathematical processing of Eq. (2) uses inadmissible transformation that yields the wrong result. Equation (7) is received by processing with limits that acceptable by mathematical rules. For the practical application is preferable to use Eq. (7) which is correct in mathematical modeling for the radial acceleration of rotating objects. The known expression for the radial acceleration Eq. (2) gives too high value than the real one. The mechanism computed by this value has excessive safety factors and sizes. The expression (Eq. (7)) gives less result in computing and is important for the engineering practice.

Conclusion

The conducted analysis of the radial accelerations for the rotating objects with the variable angular velocity about a fixed point showed several analytical solutions. The new mathematical model for the radial accelerations gives an exact solution and known ones contain vague assumptions. The engineering should use the exact expression of the radial acceleration of a rotating object about a fixed point. Computing machine components by correct mathematical models for the radial acceleration results in their high quality. In that regard, this is also a good example of the educational processes of classical mechanics.

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