Mini Review ISSN 2767-3901

International Journal of Theoretical & Computational Physics

Analytical Study of the Behavioral Trend of Charged Particle Interacting with Electromagnetic Field: Klein-Gordon/Dirac Equation

Emmanuel Ifeanyi Ugwu[1*], Usman Rilwan[1] and Rasaq Bello[2]

[1]*Department of Physics, Nigerian Army University, No.1 Biu-Gombe Road, P.M.B 1500, Biu, Nigeria*

[2]*Department of Physics, Federal University of Kashere, Gombe, Nigeria*

***Correspondence author**
Emmanuel Ifeanyi Ugwu
Department of Physics
Nigerian Army University
Nigeria

Submitted : 2 Feb 2022 ; Published :5 July 2022

Citation: Emmanuel U., Usman R., Rasaq B. Analytical Study of the Behavioral Trend of Charged Particle Interacting with Electromagnetic Field: Klein-Gordon/Dirac Equation. I J T C Physcics, 2022; 3(3): 1-3.

Abstract
The analysis of the behavioral trend of particle using Klein-Gordon and Dirac equation that was minimally coupled to electromagnetic wave four-vector potential has been carried out. In the analysis, it was clearly observed that each of them has non-relativistic limit at one stage or the other and based on this limitation, there is a great challenge posed on the idea of single particle interpretation since in each case there is a particle and an anti-particle. It therefore reveals the fact that there is no conceptually real existence of single particle in isolation when it comes to relativistic quantum mechanics for any of the equations being used to study the interaction of particle with electromagnetic field.

Keywords: Analysis, charged particle, interaction, hehaviour, Electromagnetic field, Relativistic Quantum mechanics, Couple. Potential.

Introduction
Since the advent of quantum mechanics, the study of particle behaviour in different media especially the interaction of particle with electromagnetic has been of great interest in both non-relativistic and non-relativistic domain. This further led Gordon and Dirac to go ahead in formulation of their own equation respectively Akhiezer and Berestetskii (1965), Greiner (1987) which has revolutionalize the study of particle in a field that has brought about quantum electrodynamics thereby paving the ways for the use their equation for analysis of behavior of particle in E.M field (Simulk & Krivs ki, 2014; Frandkin et al., 1991; Ugwu & Echi, 2013). For instance, a new class of exact solutions of Klein-Gordon equation for a charge particle in electromagnetic wave medium using laser field as case study has been examined in which the analytic solutions of the particle interacting with field was considered (Fedorov, 1997). It was found that the best example within the family of such solutions are the volkov states of electron or the particle using exact solution of Dirac, Klein-Gordon or Schrodinger equation(Boca & Florescu, 2011; Keldish, 1964; Bunkin & Fedorov, 1965). The solution from their work have been used for various field analysis (Brown & Kibble, 1964; Eberly, 1969; Magagzev, 2012; Bagrov & Gitman, 1990; Bagrov & Gitman, 2014). It was clearly observed that the results that emerged from their solutions and the analysis revolutionalized the applications of Klein-Gordon and Dirac equation in relativistic and non-relativistic quantum mechanics which has invariably increased the understanding of behavior of particles interacting with electromagnetic field and their solutions could be expressed in a closed form for a quantized plane wave Berson and Valdmanis (1973) and be applied in generalization of a quantized state (Berson & Valdmanis, 1973; Fedorov & Kazakov, 1973) . Relativistically both equations has been observed to be a good tool for the study of charged particle in external E.M field respectively with a specific interpretation of the particle in terms of wave equation describing either Bosons or Fermions having spin zero or spin half and may be showcased from the result of their solution when they interact with external electromagnetic field (Berson & Valdmanis, 1973; Simulk & Krivs ki, 2014; Sitenko & Yushchenko,2014). It was also noted that each of them in attempting to get its solution was coupled with one form of electromagnetic wave potential or the other before any applicable method of solution especially separation of variable the semi-classical and quantum mechanical frames for any particle in the electromagnetic field (Becker, 1977; Breev & Shpapovalov, 2016).

In this work however, we intend to study the behavior trend of charged particle interacting with electromagnetic field analytically using Klein- Gordon and Dirac equation coupled with electromagnetic four-vector potential and to ascertain the state of single particle interaction with electromagnetic field.

Mathematical supplement

The mathematical representation of both Klein-Gordon and Dirac equation and their minimal coupling with electromagnetic potential are to be presented here as it will enable a good presentation of the behavior of charged particle in the duo respectively with the picture of particle's behavior in each of them.

Klein- Gordon Equation

To deduce Klein-Gordon equation in order to analyze the behavior of charged particle in Klein-Gordon equation, the free particle Klein-Gordon equation has to be minimally coupled with four- vector potential to enable deduction of Klein-Gordon equation involving electromagnetic field in which we obtain

$$\left[g^{\mu\nu}\left(i\hbar\frac{\partial}{\partial x^{\nu}}-\frac{e}{c}A_{\nu}\right)\left(i\hbar\frac{\partial}{\partial x^{\mu}}-\frac{e}{c}A_{\mu}\right)\right]\psi=m_o^2c^2\psi \quad (1)$$

This equation can be subsequently used to characterize the charge in terms charge and current densities respectively which will invariably result correspondingly to four-current densities in the E.M field as given below,

$$j_{\nu}=\frac{i\hbar e}{2m_o}\left[\psi^*\frac{\partial}{\partial x^{\mu\nu}}\psi-\psi\frac{\partial}{\partial x\mu}\right]-\frac{e^2}{m_oc}A_{\mu}\psi\psi^*=\{c\rho'-j'\} \quad (2)$$

Which otherwise agrees with continuity equation for four-current density that depicts conservation law as in relation below

$$g^{\mu\nu}\frac{\partial}{\partial x^{\mu}}j_{\nu}=\frac{\partial}{\partial x^{\mu}}j\mu \quad (3)$$

With suitable normalization, of the same equation (1), expression for charge conservation could be obtained explicitly as

$$\rho'=\frac{i\hbar e}{2m_oc^2}\left(\psi*\frac{\partial}{\partial t}\psi-\psi\frac{\partial}{\partial}\psi*\right)-\frac{e^2}{m_oc}A_o\psi\psi^* \quad (4)$$

$$j'=-\frac{i\hbar e}{2m_o}\left[\psi\nabla\psi*-\psi^*\nabla\psi\right]-\frac{e^2}{m_oc}A\psi\psi^* \quad (5)$$

It is obvious that in equations (4) and (5), electromagnetic potential appeared with opposite charge sign in the duo, one representing electron and the other the opposite particle of electron. This analytically presents a difficult task in interpreting the concept of single particle when it come the study of the interaction of particle with electromagnetic field using Klein- Gordon equation.

Dirac Equation

With Dirac equation, Hamiltonian involving Dirac particle in relation to its interaction with electromagnetic field has to be deduced using electromagnetic four- potential just like that of Klein-Gordon counterpart and minimally couple it into Dirac equation for free particle.

The four potential is

$$A^{\mu}=\{A_o(x),\mathrm{A}(x)\} \quad (6)\text{a}$$

$$i\hbar\frac{\partial\psi}{\partial t}=\left[\frac{\hbar c}{i}\left(\hat{\alpha}_1\frac{\partial}{\partial x^1}+\hat{\alpha}^2\frac{\partial}{\partial x^2}+\hat{\alpha}^3\frac{\partial}{\partial x^3}+\hat{\beta}m_oc^2\right)\right]\psi\equiv\hat{H}_1\psi \quad (6)\text{b}$$

Where $\alpha_1,\alpha_2,\alpha_3$ are unknown coefficients And then with minimal coupling of

$$\hat{p}^{\mu}\rightarrow\hat{p}^{\mu}-\frac{e}{c}\mathrm{A}^{\mu}\equiv\hat{\Pi}^{\mu} \quad (7)$$

Dirac equation is now introduced with electromagnetic potential that transforms it to

$$c\left(i\hbar\frac{\partial}{\partial ct}-\frac{e}{c}A_o\right)\psi=\left[c\hat{\alpha}-\left(\hat{p}-\frac{e}{c}A\right)+\hat{\beta}m_oc^2\right]\psi \quad (8)$$

Representing the Dirac equation involving electromagnetic field and from where the Hamiltonian emerges as

$$\hat{H}=c\hat{\alpha}.\left(\hat{P}-\frac{e}{c}A\right)+\beta m_oc^2+e\phi \quad (9)$$

Where the equation of motion of an arbitrary operator $\hat{F}$ involves position operator which contains Hamiltonian is given as

$$\frac{d\hat{F}}{dt}=\frac{\partial\hat{E}}{\partial t}+\frac{i}{\hbar}\left[\hat{H},\hat{F}\right] \quad (10)$$

With the corresponding position operator given as

$$\frac{\partial\hat{x}}{\partial t}=\frac{i}{\hbar}\left[\hat{H},\hat{x}\right] \quad (11)$$

Since $\frac{\partial\hat{x}}{\partial t}$ is zero, then

$$\left[\hat{H},\hat{x}\right]=c\left[\hat{\alpha}.\hat{p},\hat{x}\right]_{-}-e\left[\hat{\alpha}.\mathrm{A},\hat{\mathrm{x}}\right]+m_oc^2\left[\hat{\beta},\hat{x}\right]_{-}+e\left[\phi,\hat{x}\right] \quad (12)$$

Where the coulomb potential and this is implies that $[\phi,\hat{x}]=0$

And as $\left[\hat{\alpha}.\hat{A},\hat{\mathrm{x}}\right]=0$, it connotes that $\hat{x}$ commutes with $\hat{\alpha}$ as well commute with A It now leads to

$$\left[\hat{H},\hat{x}\right]_{-}=\left(\frac{\hbar}{i}\right)c \quad (13)$$

Hence, $\frac{d\hat{x}}{dt}=c\hat{\alpha}=\hat{v}$ (14)

Showing that the velocity of a Dirac particle is as given in equation (14)

Thus considering the action of the operator on single component of Dirac particle, we obtain

$$\hat{v}^j\psi=c\hat{\alpha}_j\psi=\pm c\psi \quad (15)$$

Since it is seen that the operator $\hat{\alpha}$,has eigenvalues ±1 ie

$\hat{\alpha}_j = \pm 1$, it agrees with the report that Dirac particles always move with speed of light meaning that it is not as same as that of classical analogy in which particles are distinguished in terms of even/ true particle or odd particle i.e. negative or positive particle.

Analytical Discussion

It is clear as observed from the analysis that in using both Klein-Gordon and Dirac equation in study of particle behavior and interaction with electromagnetic field that primarily, in the formulation of any of the equation, it has to be first of all coupled minimally to electromagnetic four-vector potential (Greiner, 1987) before it is applied in the study as seen in equations (1) and (6) . Similarly, equations (5) and (6) had electromagnetic potential that appeared with opposite charge sign in the duo, one representing electron and the other the opposite particle of electron as presented from Klein-Gordon equation. While in the case of Dirac who presented his own case in terms of operator operating on a single particle, it was also observed that the eigenvalues of the operator was given as . This result invariably indicates that there is existence of particle and an anti-particle. The analysis also indicated that the particle exhibited a non –relativistic limit in their behavior at one time in terms of particle behavioural trend which calls for a relativistic interpretation for proper understanding from that analysis, both agreed that there is a particle and an antiparticle. Though it may be claimed that in case of Klein-Gordon's analysis, the idea is based on inference Greiner 1987; Ugwu ,2021), but however there is this difficult in a single particle interpretation in both as far as relativistic quantum mechanics is concerned.

References

1. Akhiezer, A.I. & Berestetskii, V.B. (1965). Quantum Electrodynamics, *Dover Science Publisher*.
2. Greiner ,W. (1987). Relativistic Quantum Mechanics, wave equations, *Springer*, Verlag.
3. Simulk, V.M & Krivs ki, I. Yu. (2014). Link between the Relativistic canonical quatum mechanics and the Dirac equation, *Uni J. Phys. Appl.*, 2, 115.
4. Frandkin, E.S, Gitman, D.M & M.Sh (1991). Quantum Electrodynamic with Unstable Vacuum, *Spriger* Verlag, berlin Heidelberg,
5. Ugwu, E.I. & Echi, M.I (2013). Analytical Study of Band Structure of Material using Relativistic Concept. *Journal of Applied Mathematics, 4*(9), 1287-1289.
6. Fedorov, M.D. (1997). Atomic and free Electron in a Srong laser field, *world Scientific*, Singgapore.
7. Boca, M. & Florescu, V. (2011). On the properties of the Volkov solutions of the Klein- Gordon equation *J.Phys.A;Math.Phys.,*14, 1481-1484.
8. Keldish, L.V. (1964). Ionization in the field of strong electromagnetic wave Zh. Ekp Teor Fiz(U.S.S R) 47 1945-1947 [Sov. Phys. JETP 20 1307-1314 (1965)]
9. Bunkin, F.E & Fedorov, M.V. (1965). Bremsstralung in a stong radiation field .Zh.EKSP. fiz(U.SS.R) 49 1215-1221 [Sov.Phys.JETP 22844-847 (1966)
10. Brown, L.S. & Kibble,T.W.B. (1964). Interaction of intense laser beams withelectrons. *Phys. Rev, 133*, 705-719.
11. Eberly, J.H. (1969). Interaction of very intense light with free electrons. *Progree in Optics VII*(Ed.E.Wolf), 359-415.
12. Magagzev, A.A (2012). Integration of Klein-Gordon equation-Fock equations in an external electromagnetic field on Lie group *Theor.Math.Phys*., 33, 1654.
13. Bagrov, V,G. & Gitman, D.M. (1990). Exact solution of relativistic, Wave Equation Dordrecht.
14. Bagrov, V. G & Gitman, D.M (2014). The Dirac Equation and its solution, Boston; De Gruyter
15. Berson, I.Y. & Valdmanis, J. (1973). Electron in the two field of two monochromatic electromagnetic waves. *J. Math. Phys,* 14, 141481-1484.
16. Fedorov, M.V. & Kazakov, A.E. (1973). An electron in a quantized plane planes wave and in a constant magnetic field *.Zeitschrift fur Physik,* 261, 191-202.
17. Berson, I.Y. & Valdmanis, J. (1973). Electron in the two field of two monochromatic electromagnetic waves. *J. Math. Phys*, 14, 141481-1484.
18. Simulk, V.M & Krivs ki, I. Yu (2014). Link between the Relativistic canonical quatum mechanics and the Dirac equation, *Uni J. Phys. Appl*, 2, 115.
19. Sitenko, Yu. & Yushchenko, S.A. (2014). The Casimir effect with quantized charged scalar matter in background magnetic field. *Int J. Mod Phy,* A 29, 1450052.
20. Becker, W. (1977). Relativistic charged particles in the field of an electromagnetic plane wave medium. *Physica*, A 87, 601-613.
21. Breev, A.I, & Shpapovalov, A.V (2016). The Dirac Equation in an external electromagnetic Field: Symmetry algebra and exact Integration, *Journal of Physics; Conference Series,* 670, 012115.