Research Article ISSN 2832-9384

Journal of Materials and Polymer Science

Novel Materials Based on Nonideal Supercrystal Formed by a Tunnel Connected Array of Microcavities Containing Ensembles of Quantum Dots

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Submitted : 2 Aug 2022 ; Published : 20 Sept 2022

Citation: Rumyantsev, V. V. & Fedorov, S. A. Novel Materials Based on Nonideal Supercrystal Formed by a Tunnel Connected Array of Microcavities Containing Ensembles of Quantum Dots. J mate poly sci, 2022; 2(3): 1-8.

Abstract

Numerical model for a defect-containing lattice of microcavities with embedded ultracold atomic clusters (quantum dots) is developed. It is assumed that certain fractions of quantum dots are absent, which leads to transformation of polariton spectrum of the overall structure. The dispersion relations for polaritonic modes are derived as functions of structure defects concentrations and elastic strain. It is shown that, as a result of elastic strain of the system and presence of structural defects under study, it is possible to achieve necessary changes in its energy structure (and, therefore, optical properties) determined by the rearrangement of the polariton spectrum.

Keywords: one-dimensional microcavity lattice, quantum dots, electromagnetic excitations, structure defects, uniform elastic deformation

Introduction

The important features of photonic band-gap structures under discussion (Milonni, 2005) are connected with 'slow' light, which is one of the promising fundamental physical phenomena that can be explored in the design of various quantum optical storage devices. In particular, the effective reduction of the group velocity demonstrated in the associated optical waveguide resonators (Yang et al., 2005; Gersen et al., 2005) as well as in the different types of solid-state semiconductor multilayer structures (Turukhin et al., 2002). Key role in reducing the group velocity in these systems is played by so-called light and dark polaritons, which are linear superposition of photon states of the external electromagnetic field and the macroscopic (coherent) perturbations of two-level atomic medium.

In atomic systems, the lifetime of polaritons limited by lifetime of the excited atoms and is usually characterized by nanoscale (Vogl & Weitz, 2008). The present level of development of nanotechnologies and nanophotonics makes it possible to study the "slow" light and the phase transitions of polaritons by creating an array of coupled microcavities containing two-level atoms (Aoki et al., 2006; Hartmann et al., 2006; Zhou et al., 2007). Technologically, the data structures can be obtained based on photonic crystals with defects as microcavities doped with two-level atoms (Joannopoulos et al., 2008).

In the context of this class of problems, a spatially periodic atomic structure - polaritonic crystal formed by ensembles of atomic clusters (quantum dots) interacting with the localized electromagnetic field in a tunnel connected array of microcavities is proposed in the paper. A remarkable feature of this structure is the possibility of localization of polaritons, which is similar to the possibility of localization of light in photonic crystals in nonlinear optics (see, eg, [9]) or the localization of excitons in quasi-periodic structures in solid state physics (Albuquerque & Cottam, 2004).

Based on the representations of the ideal photonic structures developed previously (Alodjants et al., 2010), the non-ideal system of this class - the polaritonic crystal with the atomic subsystem containing the impurity atom clusters is considered in the paper. In this context, a rapidly developing research sub-area is the photonics of imperfect structures. Some of our previous works have been devoted to the design of multi-microcavity structures (Rumyantsev et al., 2014) where the dispersion of photon modes may be altered by introduction of a defect in the photonic supercrystal (Rumyantsev et al., 2014; Rumyantsev et al., 2016; Rumyantsev et al., 2018; Rumyantsev, 2018). For applications, the structural defects in super crystals are less practical than temporary defects introduce by application of external fields or strain. In the present work we consider the effect of a uniform elastic strain

on one-dimensional arrays of microcavities with embedded quantum dots. This system combines advantages of an extreme optical non-linearity provide by the coupling of quantum dots to photonic modes and the high sensitivity of the optical eigen-modes to the applied strain. We focus on particular realization of topologically ordered micropores system composed by tunnel-coupled micropores containing and without quantum dots (atomic cluster). Such systems have a high potentiality for applications in optical integrated circuits.

Theoretical Background

Basing on the approach developed in Refs. (Rumyantsev et al., 2014; Rumyantsev et al., 2014; Rumyantsev et al., 2016; Rumyantsev et al., 2018; Rumyantsev, 2018), let us consider the dispersion of optical Eigen modes in the most general case of a microcavity supercrystal composed of sublattices. Each of tunnel-coupled microcavities is assumed to confine a single optical mode. The assumption of a low density of excited states of structural elements in the resonator and atomic subsystems makes it possible to describe the quadratic part $\hat{H}^{ex}$ of Hamiltonian, which describes elementary excitations in a microcavity chain (containing quantum dots or otherwise) within the Heitler-London approximation (Agranovich, 1968). In the one-level model, taking into account the uniform elastic deformation in the system, $\hat{H}^{ex}$ is dependent on the deformation tensor that is sensitive to the applied strain. Adapting the Heitler-London approximation and a single-level model, the Hamiltonian $\hat{H}^{ex}(\hat{\varepsilon})$ can be written as:

$$\hat{H}^{ex}(\hat{\varepsilon}) = \sum_{\substack{\mathbf{n},\mathbf{m},\alpha,\beta,\\ \lambda,\sigma}} D^{\lambda\sigma}_{\mathbf{n}\alpha,\mathbf{m}\beta}(\hat{\varepsilon})\hat{\Phi}^{+}_{\mathbf{n}\alpha\lambda}\hat{\Phi}_{\mathbf{m}\beta\sigma} = \sum_{\substack{\alpha,\beta,\lambda,\sigma\\ \mathbf{k}}} D^{\lambda\sigma}_{\alpha\beta}(\mathbf{k},\hat{\varepsilon})\hat{\Phi}^{+}_{\alpha\lambda}(\mathbf{k})\hat{\Phi}_{\beta\sigma}(\mathbf{k}) \tag{1}$$

where

$$D^{11}_{\mathbf{n}\alpha,\mathbf{m}\beta}(\hat{\varepsilon}) = \hbar\omega^{at}_{\mathbf{n}\alpha}\delta_{\mathbf{n}\alpha,\mathbf{m}\beta} + V_{\mathbf{n}\alpha,\mathbf{m}\beta}(\hat{\varepsilon}),\ D^{22}_{\mathbf{n}\alpha,\mathbf{m}\beta} = \hbar\omega^{ph}_{\mathbf{n}\alpha}\delta_{\mathbf{n}\alpha,\mathbf{m}\beta} - A_{\mathbf{n}\alpha,\mathbf{m}\beta}(\hat{\varepsilon}),$$

$$D^{12}_{\mathbf{n}\alpha,\mathbf{m}\beta}(\hat{\varepsilon}) = D^{21}_{\mathbf{n}\alpha,\mathbf{m}\beta}(\hat{\varepsilon}) = g_{\mathbf{n}\alpha}(\hat{\varepsilon})\delta_{\mathbf{n}\alpha,\mathbf{m}\beta}$$

$$\hat{\Phi}^{\lambda=2}_{\mathbf{n}\alpha} = \hat{\Psi}_{\mathbf{n}\alpha},\ \hat{\Phi}^{\lambda=1}_{\mathbf{n}\alpha} = \hat{B}_{\mathbf{n}\alpha} \tag{2}$$

In Eqs.(1,2) $\omega^{ph}_{\mathbf{n}\alpha}$ is the frequency of the photonic mode localized in the **n** α-th lattice site (microcavity), $\hat{\Psi}^{+}_{\mathbf{n}\alpha}, \hat{\Psi}_{\mathbf{n}\alpha}$ are bosonic creation and annihilation operators for this mode written in the node representation, $\hbar\omega^{at}_{\mathbf{n}\alpha}$ is excitation energy of the quantum dot in the **n** α-th lattice site, $\hat{B}_{\mathbf{n}\alpha}, \hat{B}^{+}_{\mathbf{n}\alpha}$ are creation and annihilation operators of quantum dot excitons, $A_{\mathbf{n}\alpha\mathbf{m}\beta}(\hat{\varepsilon})$ is the matrix of resonance interaction, which describes an overlap between optical fields of resonators in the **n** α-th and **m** β-th lattice sites and hence defines the jump probability of the corresponding electromagnetic excitation, $V_{\mathbf{n}\alpha\mathbf{m}\beta}(\hat{\varepsilon})$ is the matrix of resonance interaction between quantum dots embedded in the **n** α-th and **m** β-th lattice sites, is the matrix of resonance interaction between quantum dot in the **n** α--th lattice site and electromagnetic field localized at the same site. Values 1 and 2 of indices indicate, λ, σ respectively, the presence or absence of quantum dots in corresponding cavities.

In the right-hand side expression of Eq. (1) (summation over **k**) matrices $D^{\lambda\sigma}_{\alpha\beta}(\mathbf{k},\hat{\varepsilon})$ and $\Phi_{\alpha\lambda}(\mathbf{k})$ have the forms

$$D^{\lambda\sigma}_{\alpha\beta}(\mathbf{k},\hat{\varepsilon}) = \sum_{\mathbf{m}} D^{\lambda\sigma}_{\mathbf{n}\alpha\mathbf{m}\beta}(\hat{\varepsilon})\exp\left[i\mathbf{k}\cdot\left(\mathbf{r}_{\mathbf{n}\alpha} - \mathbf{r}_{\mathbf{m}\beta}\right)\right]$$

and

$$\hat{\Phi}_{\alpha\lambda}(\mathbf{k}) = \frac{1}{\sqrt{N}}\sum_{\mathbf{n}}\hat{\Phi}_{\mathbf{n}\alpha\lambda}\exp\left(-i\mathbf{k}\cdot\mathbf{r}_{\mathbf{n}\alpha}\right)$$

(N is the number of elementary cells in the lattice). Such representation of matrices is possible due to preservation of the translation invariance of the system under the uniform strain. Let us note that the wave vector, which characterizes eigenstates of electromagnetic excitations, ranges within the first super crystal Brillouin zone, whose boundaries are in their turn functions of strain through the dielectric tensor ε.

Eigenvalues of the Hamiltonian (1) are found by its diagonalization through the Bogolyubov-Tyablikov transformation (Agranovich, 1968). This yields the following equation for elementary excitation spectrum $\Omega(\mathbf{k},\hat{\varepsilon})$:

$$\det\left\|D^{\lambda\sigma}_{\alpha\beta}(\mathbf{k},\hat{\varepsilon}) - \hbar\Omega(\mathbf{k},\hat{\varepsilon})\delta_{\alpha\beta}\delta_{\lambda\sigma}\right\| = 0 \tag{3}$$

On the basis of this equation below we investigate in detail the spectrum of exciton-polariton modes in a non-ideal lattice of tunnel-coupled micropores with embedded quantum dots.

Results and Discussion

Exciton-like excitations in a non-ideal one-dimensional microcavity lattice.

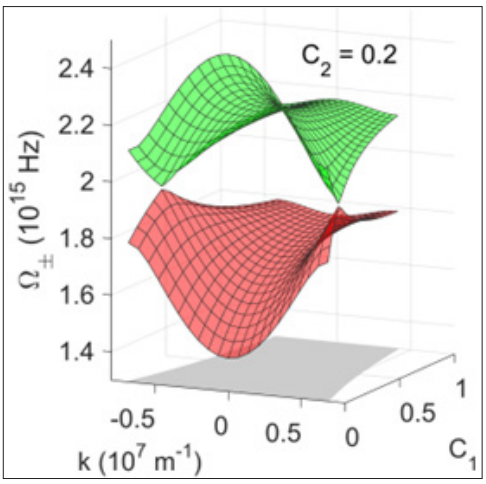
Let us first consider electromagnetic excitations (the so-called exciton-like excitations [13]) localized in a two-sublattice one-dimensional lattice of a tunnel connected microcavities without quantum dots (in this case $D^{12}_{n\alpha,m\beta} = D^{21}_{n\alpha,m\beta} = 0$).

In the absence of elastic deformation of the lattice within the nearest-neighbour approximation the corresponding spectrum $\omega(k)$ follows from relations (1)-(3):

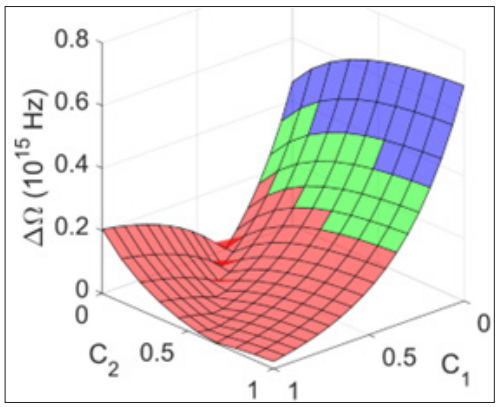
$$\begin{vmatrix}\hbar\omega^{ph}_{1} - \hbar\Omega(k,C_1,C_2) & -A_{12}(k,C_1,C_2)\\ -A_{21}(k,C_1,C_2) & \hbar\omega^{ph}_{2} - \hbar\Omega(k,C_1,C_2)\end{vmatrix} = 0 \tag{4}$$

Numerical evaluation of the quantities, which define the spectrum peculiarities shall be performed for the values of resonant photonic modes localized at lattice sites

$\omega^{ph}_{1} = 2\pi\times 311\,THz$ and $\omega^{ph}_{2} = 2\pi\times 331\,THz$



(a)



(b)

Figure 1: Concentration dependence $\Omega_{\pm}(k,\ C_1,C_2)$ for : a) C_1=0,3 and b) concentration dependence of the band gap width $\Delta\Omega(C_1,C_2)$.

Similarly, to Ref. (Rumyantsev et al., 2018) it is assumed that $A_{12}\left[a_1(0)\right]/2\hbar = 3{,}5\cdot10^{14}\,Hz$, $A_{12}\left[a_2(0)\right]/2\hbar = 1{,}2\cdot10^{14}\,Hz$,

$V_{11}/2\hbar = 1\cdot10^{13}\,Hz$, $a_1(0) = a_1^{(1)} = 1\cdot10^{-7}\,m$

$a_2(0) = a_2^{(1)} = 3\cdot10^{-7}\,m$ where $d(0) = a_1(0) + a_2(0)$. Numerically computed surfaces in Fig.1a describe the dispersion dependence of frequencies $\Omega_{\pm}(k,C_1,C_2)$ of the studied collective excitations. We remind that ranges within the first Brillouin zone $-\frac{\pi}{d(C_1,C_2)} < k < \frac{\pi}{d(C_1,C_2)}$. Fig.1b depicts concentration dependence of the corresponding band gap width $\Delta\Omega(C_1,C_2) \equiv \min_k\left[\Omega_+(C_1,C_2) - \Omega_-(C_1,C_2)\right]$.

Polaritonic excitations in a one-dimensional two-sublattice non-ideal microcavity lattice

Basing on the general theory developed in Section 2 let us proceed to consider quasi-particle (polaritonic) excitations in a two-sublattice one-dimensional microcavity lattice (see Fig. 2) with same-type quantum dots embedded in one of the sublattice (e.g. in the first one, i.e. $\alpha = \beta = 1$).

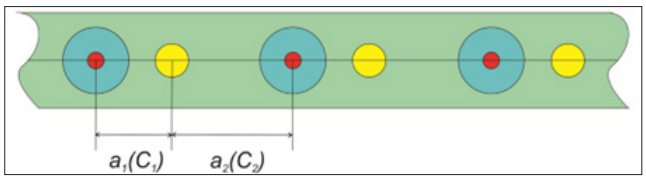


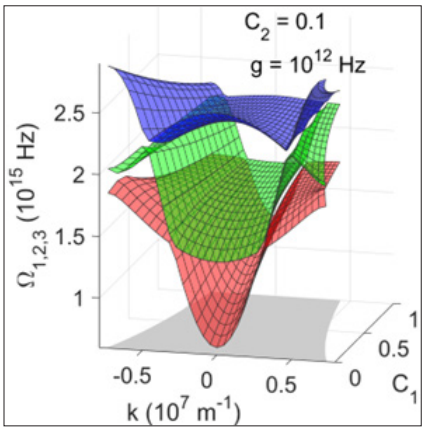
Figure 2: Schematic of the non-ideal two-sublattice one-dimensional microcavity array with quantum dots embedded in the first sublattice

In the absence of elastic deformation of the lattice (within the nearest neighbour approximation) the relations (1)-(3) yield the following equation for the elementary excitation spectrum $\Omega(k)$:

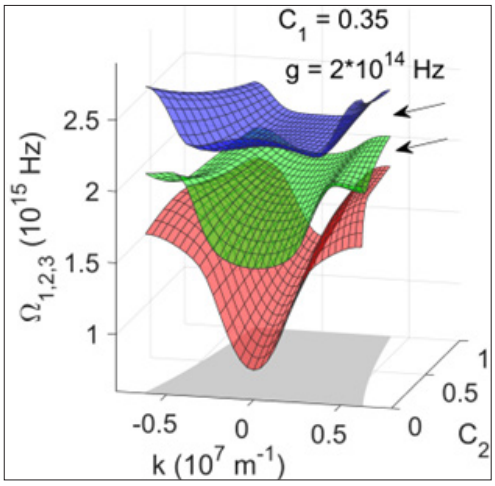
$$\begin{Vmatrix} \hbar\ {}_1^{at} - V_{11}(k) - \hbar\Omega(k) & 0 & g_1 & 0 \\ 0 & -\Omega(\ ) & 0 & 0 \\ g_1 & & \hbar\ {}_1^{ph} - \hbar\Omega(k) & -A_{12}(k) \\ 0 & 0 & -A_{21}(k,) & \hbar\ {}_2^{ph} - \hbar\Omega(k) \end{Vmatrix} \quad (5)$$

Since composition of quantum dots is not being varied the parameter of resonant interaction between a quantum dot and electromagnetic field localized at a same site is always the same. Calculation of quantities, which define the spectrum shape of polaritonic excitations was performed for the above data and excitation frequency of quantum dots $\omega_2^{at} = 2\pi\cdot202\ THz$. Also similarly to Ref. (Rumyantsev et al., 2018) we put $V_{11}/2\hbar = 1\cdot10^{13}\,Hz$, $g_1/\hbar = 5\cdot10^{12}\ Hz$. Fig. 3a, b shows surfaces, which describe the dispersion dependence of polaritonic frequencies $\Omega_{1,2,3}(k,C_1,C_2)$ in the two-sublattice microcavity array with quantum dots embedded in one of the sublattice (surfaces are numbered bottom-up). The wave number k ranges as always within the first Brillouin zone $-\frac{\pi}{d(C_1,C_2)} < k < \frac{\pi}{d(C_1,C_2)}$ (shaded region in the plane $(k,C_{1(2)})$ in Figs. 3a, b).

Let us note that the presence of local minima at $k \neq 0$ in the dispersion surface $\Omega_3(k,C_1,C_2)$ in Figs. 3 (as well as in Fig. 2) indicates the possibility of existence (for certain defect concentrations) of Bose-Einstein polaritonic condensate for non-zero k's (in addition to Bose-Einstein condensation at k=0 at the corresponding minima in surfaces $\Omega_{1,2}(k,C_1,C_2)$).



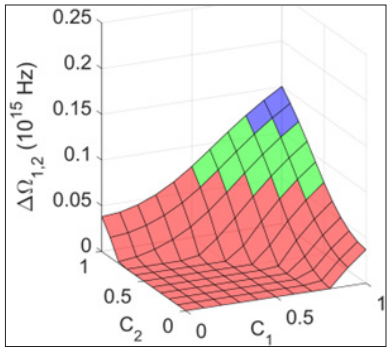
(a)



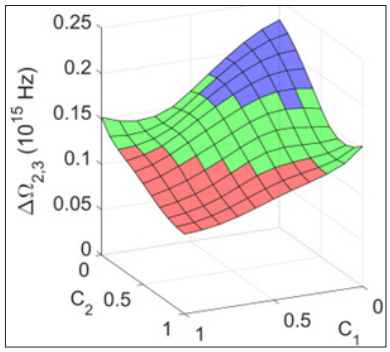
(b)

Figure 3: Dependence of polariton dispersion $\Omega_{1,2,3}(k,C_1,C_2)$ on structural defect concentration plotted for various values of parameter $g \equiv g_1/\hbar$ responsible for the resonant interaction between a quantum dot and electromagnetic field localized at a same site (arrows indicate the effect of changing g on the width of the so-called "bottle neck"

The band gap widths of polaritonic spectrum $\Delta\Omega_{12(23)}(C_1,C_2) \equiv \min_k\left[\Omega_{2(3)}(C_1,C_2)-\Omega_{1(2)}(C_1,C_2)\right]$ are plotted as functions of concentrations of structural defects in Figs. 4a, b.



(a)



(b)

Figure 4: Dependences $\Delta\Omega_{12(23)}(C_1,C_2)$ of the band gap widths on structural defect concentrations: a) $\Delta\Omega_{12}(C_1,C_2)$, b) $\Delta\Omega_{23}(C_1,C_2)$.

Exciton-like excitations in a one-dimensional two-sublattice microcavity array under a uniform elastic deformation

To develop our model let us consider a one-dimensional microcavity lattice subjected to elastic stress (extension or compression) directed along the chain. Under a uniform deformation described by tensor each cavity changes its position and so the lattice constant $d(\varepsilon)$ can be written as:

$$d(\varepsilon) = (1+\varepsilon)d_0, \tag{6}$$

where d_o is the lattice constant of a strain-free structure, and is the corresponding component of tensor ε̇. The necessary for finding the electromagnetic spectrum reciprocal lattice constant b(ε) can therefore be obtained from the obvious relation:

$$b(\varepsilon)\cdot d(\varepsilon) = 2\pi \tag{7}$$

In what follows we shall assume that the microcavity array is constituted by two sublattices. Position of microcavities is defined by the equality $r_{n\alpha}(\varepsilon) = r_n(\varepsilon) + r_\alpha(\varepsilon)$, and therefore their positions in the zeroth cell of the first and second sublattices ($r_{n=0}=0$) are, correspondingly: $r_{01}=0$ and $r_{02}(\varepsilon)=a(\varepsilon)$. The spectrum of exciton-like excitations $\Omega(k,\varepsilon)$ is found from relation (3):

$$\begin{Vmatrix} \hbar\Omega(k,\varepsilon)-\hbar\omega_1^{ph} & A_{12}(k,\varepsilon) \\ A_{21}(k,\varepsilon) & \hbar\Omega(k,\varepsilon)-\hbar\omega_2^{ph}(\varepsilon) \end{Vmatrix} = 0 \tag{8}$$

Quantities $A_{\alpha\beta}(k,\varepsilon)$ in Eq. (8) are Fourier-transforms of matrix $A_{n\alpha m\beta}(\varepsilon)$ of resonance interaction: $A_{\alpha\beta}(k,\varepsilon) = \sum_m A_{n\alpha m\beta}(\varepsilon)\exp\left\{ik\left[r_{n\alpha}(\varepsilon)-r_{m\beta}(\varepsilon)\right]\right\}$. In the frames of our model and within the nearest-neighbour approximation matrix elements assume the following form:

$$A_{12}(k,\varepsilon) \equiv A_{12}[a(\varepsilon)]\exp[-ik\cdot a(\varepsilon)] + A_{12}[d(\varepsilon)-a(\varepsilon)]\exp\{-ik\cdot[d(\varepsilon)-a(\varepsilon)]\},$$
$$A_{21}(k,\hat{\varepsilon}) = A_{21}[a(\varepsilon)]\exp[ik\cdot a(\varepsilon)] + A_{21}[d(\varepsilon)-a(\varepsilon)]\exp\{ik\cdot[d(\varepsilon)-a(\varepsilon)]\} \tag{9}$$

According to Ref. (Rumyantsev et al., 2016), quantities $A_{12(21)}[a(\varepsilon)]$, which are components of matrix $A_{n\alpha m\beta}(\varepsilon)$ of resonance interaction corresponding to nearest neighbours equal to $A_{12(21)}[a(\varepsilon)] = A_{12(21)}(a)\exp(-\varepsilon)$,

$A_{12(21)}[d(\varepsilon)-a(\varepsilon)] = A_{12(21)}(d-a)exp(-\varepsilon)$. In our case we put $A_{12}(a) \approx A_{21}(a)$, $A_{12}(d-a) \approx A_{21}(d-a)$ Relation (8) shows that the dispersion law $\Omega(k,\varepsilon)$ of elementary electromagnetic excitations is determined both by frequency characteristics of resonator array and by the explicit form of $A(k,\varepsilon)$, as well as by the nature of deformation (e.g. by a uniaxial extension ε > 0 or ε < 0 contraction).

Further calculations were performed for a uniaxial deformation of a uniform isotropic one-dimensional medium. The following modelling parameters were adopted: frequencies of resonance photonic modes in cavities (independent of deformation ε) were put equal to $\omega_1^{ph} = 2\pi\times 211 THz$ and $\omega_2^{ph} = 2\pi\times 310 THz$ and $A_{12}(a)/2\hbar = 0.9\cdot 10^{14} Hz$, $A_{12}(d-a)/2\hbar = 1.2\cdot 10^{14} Hz$.

$a = 1 \cdot 10^{-7} m$, $d = 9 \cdot 10^{-7} m$ Fig. 5 shows the dependencies $\Omega_\nu(k,\varepsilon), (\nu = 1,2)$ for deformed one-dimensional lattice for various values of ε . Shaded region in the (k, ε)-plane corresponds to the first Brillouin zone.

An important property of band gap photonic structures is their ability to produce the so-called "slow light". It has important application in designing quantum optical information processing devices. The effective decrease of quasiparticle group velocity was shown to occur in coupled wave-guide optical resonators (Yang et al., 2005) and in various types of multilayer semiconductor structures (Turukhin et al., 2002). A key role in decreasing the group velocity is played by the character of quasiparticles' effective mass Fig. 6 depicts the dependence of the effective mass $m_{eff(\nu)} = \hbar \left[\frac{\partial^2 \Omega_\nu(k,\varepsilon)}{\partial k^2} \Big|_{k=0} \right]^{-1}$ of the considered exciton-like excitations on the degree of deformation. It follows from this graph that a careful choice of permits to attain the necessary parameters of the "slow light". It is often important to know how peculiarities of the spectrum are manifested in quasiparticle density of states $\rho_\nu(\Omega,\varepsilon)$, which obviously should depend on the degree of deformation (ν = 1;2). According to Ref. (Rumyantsev et al., 2014) in our case functions $\rho_\nu(\Omega,\varepsilon)$ are defined as:

$$\rho_\nu(\Omega,\hat{\varepsilon}) = \frac{d(\varepsilon)}{2\pi} \sum_i \frac{1}{\left| \frac{d\Omega_\nu(\varepsilon,k)}{dk} \right|_{k_i}} \qquad (10)$$

where k_i are the roots of equation $\Omega_\nu(k) = \Omega$.

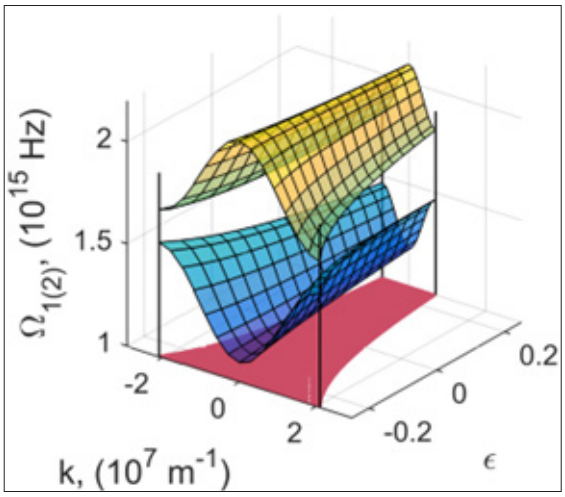


Fig.ure 5: Dispersion surfaces $\Omega_{1(2)}(k,\varepsilon)$ of the deformed two-sublattice microcavity array subjected to a uniform elastic strain

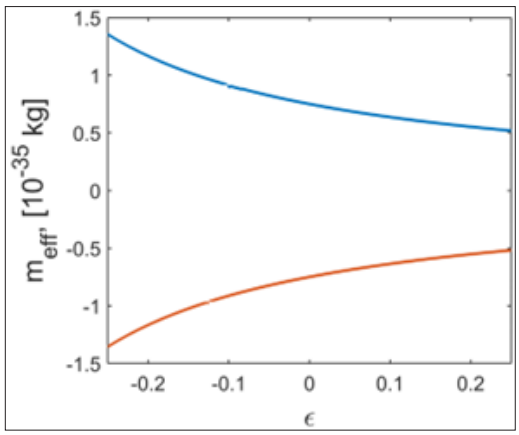


Figure 6: Dependence of the effective masses of exciton-like excitations on the degree of deformation in a two-sublattice microcavity array

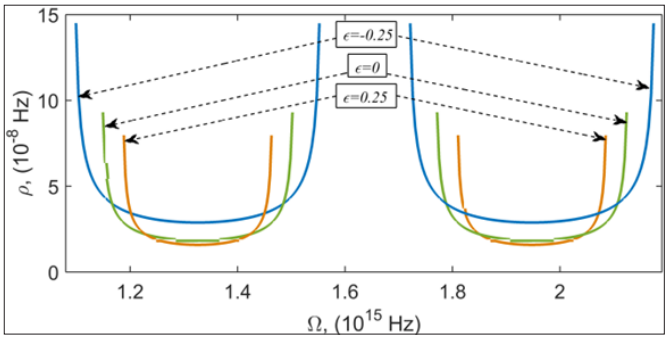


Figure 7: Variation of the density of polaritonic states $\rho_\pm(\Omega,\varepsilon)$ in the lower and upper dispersion bands in a two sublattice microcavity array under elastic deformation.

Calculation in (8) is carried out for values of wave vector falling within the first Brillouin zone. Densities of states $\rho_\nu(\Omega)$ of quasiparticles in upper and lower dispersion branches are plotted in Fig. 7 for several values of ε.

Array of microcavities containing quantum dots under a uniform elastic deformation.

As next example, let's consider polaritons in a one-sublattice quantum-dot-containing chain of unevenly spaced microcavities under a uniform elastic deformation. We consider the array of identical cavities with randomly embedded quantum dots of two types, whose concentrations are, correspondingly $C_C^{(1)}$ and $C_C^{(2)}$. It is assumed, in addition, that microcavities are unevenly spaced; namely that $C_T^{(1)}$ neighbouring pairs of cavities are separated by distance $a_1(\varepsilon)$ and the remaining $C_T^{(2)}$ pairs are separated by distance $a_2(\varepsilon)$. Here we also adopt the virtual crystal approximation (Ziman, 1979; V. F. Los, 1987) based on the diagonalization of the averaged Hamiltonian (1). The corresponding procedure yields a system of uniform linear equations, whose solvability condition is given by:

$$\begin{vmatrix} \hbar\langle\omega_n^{at}(\varepsilon)\rangle_C + \langle V(k,\varepsilon)\rangle_{C,T} - \hbar\Omega(k,\varepsilon) & \langle g_n(\varepsilon)\rangle_C \\ \langle g_n(\varepsilon)\rangle_C & \hbar\omega^{ph}(\varepsilon) - \langle A(k,\varepsilon)\rangle_T - \hbar\Omega(k,\varepsilon) \end{vmatrix} = 0 \qquad (11)$$

where $\langle \omega_n^{at} \rangle_C = \sum_{\nu=1}^{2} \omega_\nu^{at} C_C^\nu$, $\langle g_n \rangle_C = g^{(1)} C_C^{(1)} + g^{(2)} C_C^{(2)}$

(it is implied that $C_C^{(1)} + C_C^{(2)} = 1$,and hence $C_C^{(1)} = 1 - C_C^{(2)} \equiv C_C$);

$$\langle V(k) \rangle_{C,T} = \sum_{\nu,\mu=1}^{2} V^{\nu\mu}(k, \{C_T\}, \varepsilon) C_C^\nu C_C^\mu$$

$$V^{\nu,\mu}(k, \{C_T\}, \varepsilon) = \sum_m \langle V_{nm}^{\nu\mu}(\varepsilon) \rangle_T \exp\left[ikr_{nm}(\{C_T\}, \varepsilon)\right]$$

Similarly, $A(k, \{C_T\}, \varepsilon) = \sum_m \langle A_{nm}(\varepsilon) \rangle_T \exp\left[ikr_{nm}(\{C_T\}, \varepsilon)\right]$

where $r_{nm}(\{C_T\}, \varepsilon) = d(\{C_T\}, \varepsilon)(n-m)$, ($C_T^{(1)} + C_T^{(2)} = 1$, $C_T^{(1)} = 1 - C_T^{(2)} \equiv C_T$).

Angular brackets in (6) denote the procedure of configuration averaging of the microcavity array over all possible positions of cavities (index "T") and compositions of quantum dots (index "C"). $d(\{C_T\}, \varepsilon)$ is the period of the "virtual" one-dimensional microcavity lattice obtained by averaging $d(\{C_T\}, \varepsilon) = C_T^{(1)} a_1(\varepsilon) + C_T^{(2)} a_2(\varepsilon)$.

Within the nearest-neighbour approximation, the quantities $V(k, \{C_T\}, \varepsilon)$, $A(k, \{C_T\}, \varepsilon)$ can be found as:

$$\begin{bmatrix} V^{\nu\mu}(k, \{C_T\}, \varepsilon) \\ A^{\nu\mu}(k, \{C_T\}, \varepsilon) \end{bmatrix} = 2 \begin{bmatrix} V^{\nu\mu}\left[d(\{C_T\}, \varepsilon), \varepsilon\right] \\ A^{\nu\mu}\left[d(\{C_T\}, \varepsilon), \varepsilon\right] \end{bmatrix} \cos\left\{kd\left[\{C_T\}, \varepsilon\right]\right\} \tag{12}$$

It follows from (11) that the dispersion relation $\Omega(k, \{C_C, C_T\}, \varepsilon)$ of polariton modes is defined by frequency characteristics of the cavities and the dots as well as by the explicit form of expressions $A(k, \{C_T\}, \varepsilon)$ and $V^{\nu,\mu}(k, \{C_T\}, \varepsilon)$.In the framework of our model, the functions $A\left[d(\{C_T\}, \varepsilon), \varepsilon\right]$ and $V^{\nu\mu}\left[d(\{C_T\}, \varepsilon), \varepsilon\right]$ of the strain degree and the defect concentrations are assumed (for $a_2(\varepsilon) > a_1(\varepsilon)$) to be equal to:

$$\begin{bmatrix} V^{\nu\mu}\left[d(\{C_T\}, \varepsilon), \varepsilon\right] \\ A\left[d(\{C_T\}, \varepsilon), \varepsilon\right] \end{bmatrix} = \begin{bmatrix} V^{\nu\mu}(a_1|_{\varepsilon=0}) \\ A(a_1|_{\varepsilon=0}) \end{bmatrix} \exp\left[-\frac{\left| d(\{C_T\}, \varepsilon) - a_1(\varepsilon) \right|}{a_1(\varepsilon)} - \varepsilon \right] \tag{13}$$

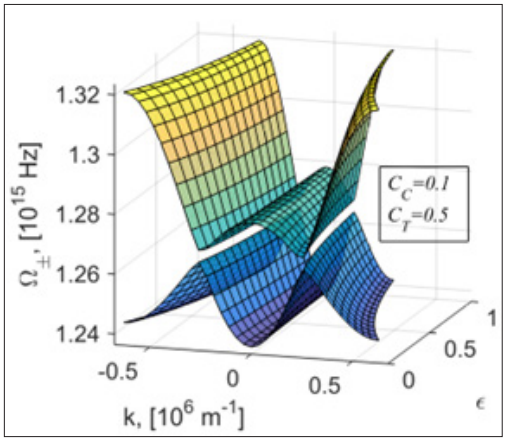
$a_1|_{\varepsilon=0} \equiv a_1$, $a_2|_{\varepsilon=0} \equiv a_2$ .Quantities $A(a_1), V^{\nu\mu}(a_1)$ characterize an overlap of optical fields of neighbouring cavities and an interaction between neighbouring quantum dots in a one-dimensional lattice with period , respectively. Such a lattice is chosen to be a reference one for the subsequent variation of distances between resonators.

The numerical calculations were carried out for the following modelling values of parameters. The frequency of cavity-localized resonance photonic modes was put equal to $\omega^{ph} = 2\pi \times 203\ THz \approx 1280 \cdot 10^{12}\ Hz$; the two types of quantum dots were assumed to be characterized by the exciton resonance frequencies and $\omega_1^{at} = 2\pi \cdot 191\ THz \approx 1200 \cdot 10^{12}\ Hz$ and $\omega_2^{at} = 2\pi \cdot 202\ THz \approx 1269 \cdot 10^{12}\ Hz$ whereas $A/2\hbar = 8 \cdot 10^{13}\ Hz$, $V^{11}/2\hbar = 1 \cdot 10^{13}\ Hz$, $V^{22}/\hbar = 3 \cdot 10^{13}\ Hz$, $V^{12} \approx V^{21} = 6 \cdot 10^{13}\ Hz$, $g^{(1)}/\hbar = 5 \cdot 10^{12}\ Hz$' $g^{(2)}/\hbar = 1.5 \cdot 10^{12}\ Hz$

(within the adopted approximation the magnitude of resonance interaction of a quantum dot with an electromagnetic field localized at the same cavity is independent of deformation). The lattice periods were set equal to $a_1 = 3.10^{-6}\ m$ and $a_2 = 7.10^{-6}\ m$. The two dispersion branches $\Omega_\pm(k, C_C, C_T)$ of the considered collective excitations in the microcavity array are plotted in Figs. 8a, b for several values of C_C and C_T. Let us remind that ranges between the values of the corresponding values:

$$-\frac{\pi}{a_2(\varepsilon) + C_T\left[a_1(\varepsilon) - a_2(\varepsilon)\right]} \le k \le +\frac{\pi}{a_2(\varepsilon) + C_T\left[a_1(\varepsilon) - a_2(\varepsilon)\right]} \tag{14}$$

Whereas C_T ranges between 0 to 1.



(a)

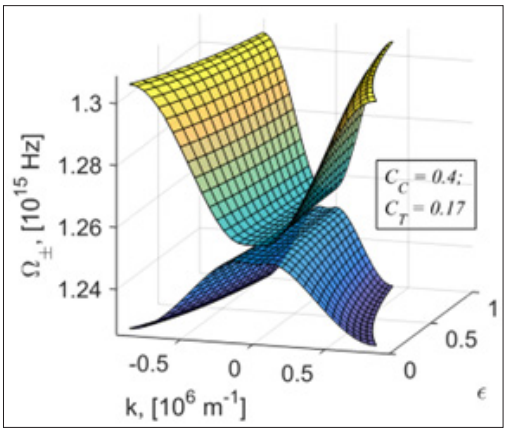


Figure 8: Dispersions $\Omega_\pm(k, C_C, C_T)$ of polaritonic excitations in a one-sublattice quantum-dot-containing chain of unevenly spaced microcavities plotted for different values of the dot concentrations C_c, C_T .

It should be noted that the shape of the dispersion curve in Fig. 5a indicates the existence of Bose-Einstein exciton condensate, where the energy minima occur for a number of states with non-zero k' s (in addition to those with $k = 0$).

Conclusion

The theoretical study of the photonic band structure of non-ideal lattices of tunnel-coupled microcavities shows that subjecting the system to the controllable elastic strain and presence of structural defects are an effective tool for altering its eigen mode structure and optical properties. This applies both for the cases of a microcavity arrays with embedded quantum dots and for quantum-dot-free lattices (Rumyantsev et al., 2014; Rumyantsev et al., 2016; Rumyantsev et al., 2018; Rumyantsev et al., 2019; Rumyantsev et al., 2022). The strain and the structural defects lead to the increase of the effective mass of the propagating photon modes in the structure and hence to the decrease of their group velocity. This results in formation of slow light mode that can be efficiently controlled by the externally applied strain. The obtained results demonstrate the possibility of controlling the group velocity of excitations, which is responsible for signaling rates in optical integrated circuits of optoelectronic devices. Numerical simulations performed on the basis of the constructed model contribute to modelling of the new class of functional porous materials, namely the so-called polaritonic systems (microcavity arrays with embedded quantum dots) where controlling of propagation of electromagnetic excitations is accomplished by an appropriate introduction of structural defects and elastic deformation.

References

1. Milonni, P. W. (2005). *Fast Light, Slow Light and Left-Handed Light*. Institute of Physics Publishing, Bristol and Philadelphia. Retrieved from https://www.fulviofrisone.com/attachments/article/411/Milonni%20P.W.%20Fast%20light,%20slow%20light%20and%20left-handed%20light%20(IOP,%202005)(ISBN%2007503
2. Yang, Z. S., Kwong, N. H., Binder, R. & Smirl, A. L. (2005). Stopping, storing, and releasing light in quantum-well Bragg structures. *Journal of the Optical Society of America B, 22*(10), 2144-2156. DOI: https://doi.org/10.1364/JOSAB.22.002144
3. Gersen, H., Karle, T. J., Engelen, R. J. P., Bogaerts, W., Korterik, J. P., van Hulst, N. F., Krauss, T. F. & Kuipers, L. (2005). Real-Space Observation of Ultraslow Light in Photonic Crystal Waveguides. *PHYSICAL REVIEW LETTERS, 94*(7), 073903. Retrieved from https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.94.073903
4. Turukhin, A. V., Sudarshanam, V. S., Shahriar, M. S., Musser, J. A., Ham, B. S. & Hemmer, P.R. (2002). Observation of Ultraslow and Stored Light Pulses in a Solid. *PHYSICAL REVIEW LETTERS, 88*(2), 023602–1. Retrieved from https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.88.023602
5. Vogl, U. & Weitz, M. (2008). Spectroscopy of atomic rubidium at 500−bar buffer gas pressure: Approaching the thermal equilibrium of dressed atom-light states. *PHYSICAL REVIEW A, 78*(1), 011401. Retrieved from https://journals.aps.org/pra/abstract/10.1103/PhysRevA.78.011401
6. Aoki, T., Dayan, B., Wilcut, E., Bowen, W. P., Parkins, A. S., Kippenberg, T. Y., Vahala, K. Y. & Kimble, H. J. (2006). Observation of strong coupling between one atom and a monolithic microresonator. *Nature, 443*(7112), 671-674. Retrieved from http://dx.doi.org/10.1038/nature05147
7. Hartmann, M. J., Brandao, F. & Plenio, M. B. (2006). Strongly interacting polaritons in coupled arrays of cavities. *Nature Physics*, 2, 849-855. Retrieved from https://www.nature.com/articles/nphys462
8. Zhou, L., Lu, J. & Sun, C. P. (2007). Coherent control of photon transmission: Slowing light in a coupled resonator waveguide doped with atoms. *PHYSICAL REVIEW A, 76*(1), 012313. Retrieved from https://journals.aps.org/pra/abstract/10.1103/PhysRevA.76.012313
9. Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. (2008). Photonic Crystals. Molding the Flow of Light, (Second Edition). *Princeton University Press, Princeton*. Retrieved from http://ab-initio.mit.edu/book/
10. Albuquerque, E. L. & Cottam, M. G. (2004). Polaritons in Periodic and Quasi periodic Structures (1st Edition). *Elsevier*, Amsterdam. Retrieved from https://www.elsevier.com/books/polaritons-in-periodic-and-quasiperiodic-structures/albuquerque/978-0-444-51627-5
11. Alodjants, A. P., Barinov, I. O. & Arakelian, S. M. (2010). Strongly localized polaritons in an array of trapped two-level atoms interacting with a light field. *Journal of Physics B: Atomic, Molecular and Optical Physics, 43*(9), 095502. https://iopscience.iop.org/article/10.1088/0953-4075/43/9/095502
12. Rumyantsev, V. V., Fedorov, S. A., Gumennyk, K. V. & Proskurenko, M. V. (2014). Peculiarities of propagation of electromagnetic excitation through a nonideal gyrotropic photonic crystal. *Physica B Condensed Matter*. 442, 57-59. DOI:10.1016/j.physb.2014.02.023
13. Rumyantsev, V. V., Fedorov, S. A., Gumennyk, K. V., Sychanova, M. V. & Kavokin, A. V. (2014). Exciton-like electromagnetic excitations in non-ideal microcavity supercrystals. *Scientific Reports, 4*, 6945. Retrieved from https://www.nature.com/articles/srep06945
14. Rumyantsev, V. V., Fedorov, S. A., Gumennyk, K. V., Sychanova, M. V. & Kavokin, A. V. (2016). Polaritons in a nonideal periodic array of microcavities. *Superlattices and Microstructures*, 89, 409-418. DOI: http://dx.doi.org/10.1016/j.spmi.2015.11.029
15. Rumyantsev, V. V., Fedorov, S. A., Gumennyk, K. V., Gurov, D. A. & Kavokin, A.V. (2018). Effects of elastic strain and structural defects on slow light modes in a one-dimensional array of microcavities. *Superlattices and Microstructures*, 120, 642-649. DOI: https://doi.org/10.1016/j.spmi.2018.06.043
16. Rumyantsev, V. V. & Fedorov, S. A. (2018). *SF J Laser Optics, 2*(2), 1-8.
17. Agranovich, V. M. (1968). Theory of Excitons. Moscow. *Nauka Publishers*.

18. Ziman, J. M. (1979). Models of disorder: The theoretical physics of homogeneously disordered systems. *Cambridge University Press*, Cambridge. Retrieved from https://catalogue.nla.gov.au/Record/2439975
19. V. F. Los'. (1987). Projection operator method in the theory of disordered systems. I. Spectra of quasiparticles. *Theoretical and Mathematical Physics*, 73, 1076-1088. DOI: 10.1007/BF01022966. Retrieved from https://link.springer.com/article/10.1007/BF01022966
20. Rumyantsev, V. V., Fedorov, S. A., Gumennyk, K. V. & Paladyan, Yu. A. (2019). Electromagnetic excitations in a non-ideal two-sublattice microcavity chain. *Physica B: Condensed Matter, 571*(15), 296-300. DOI: https://doi.org/10.1016/j.physb.2019.07.022
21. Rumyantsev, V. V., Fedorov, S. A., Gumennyk, K. V. & Rybalka, A. Ye. (2022). Polaritonic Crystal Formed by a Tunnel Connected Array of Microcavities Containing Ensembles of Quantum Dots. *International Journal of High Energy Physics, 9*(1), 13-19. Retrieved from https://www.sciencepublishinggroup.com/journal/paperinfo?journalid=124&doi=10.11648/j.ijhep.20220901.13