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Plane Strain and Stress States of Two-Layer Composite Reinforced Body in Dynamic Elastic-Plastic Formulation

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Abstract

For the design of composite and reinforced materials, a technique for solving dynamic contact problems in more precise an elastic-plastic mathematical formulation is used. To consider the physical nonlinearity of the deformation process, the method of successive approximations is used, which makes it possible to reduce the nonlinear problem to a solution of the sequences of linear problems. In contrast to the traditional plane strain, when one normal stress is equal to a certain constant value, for a more accurate description of the deformation of the sample, taking into account the possible increase in longitudinal elongation, we present this normal stress as a function that depends on the parameters that describe the bending of a prismatic body that is in a plain strain state. The problems of a plane strain and stress states of a beam made from the composite reinforced double-layer material is being solved. The reinforced or armed material consists of two layers: the upper (first) thin layer of solid steel and the lower (second) main layer of glass. Glass is a non-crystalline, often transparent amorphous solid that has widespread practical and technological use in the modern industry. Glass has high strength and is not affected by the processes of aging of the material, corrosion, and creep. In addition, this material is cheap and widely available. Glass can be strengthened, for example, in a melt quenching process. The reinforced composite beam is rigidly linked to an absolutely solid base and on which an absolutely solid impactor acts from above in the centre on a small area of initial contact.

Keywords: Plane, strain, stress, state, impact, composite, armed, reinforced, material, elastic-plastic, deformation.

Introduction

Glass is a very strong and very fragile material at the same time. The fragility of glass is due to the fact that there are many micro cracks on the surface, and when a load is applied to the glass surface, these micro cracks begin to grow and lead to the destruction of glass products. If we glue or immobilize the tops of micro cracks on the surface, we will get a strong reinforced armed material that will be lighter, stronger and not subject to degradation of material properties such as aging, corrosion and creep. The upper reinforcing layer of metal or steel can be applied to the glass surface so that metal or steel atoms penetrate deeply, fill micro cracks and bind their tops. The top layer can be quite thin. The adhesion between the layers is considered perfectly rigid. The issue of practical provision of such coupling is an important component of technological implementation. In the E.O. Paton Institute of Electric Welding of the National Academy of Sciences of Ukraine in the early 2000s, the technology of welding ceramic parts was developed. A copper membrane was clamped between two ceramic parts. A powerful electric impulse was applied to the membrane, as a result of which the copper membrane instantly evaporated and the copper atoms penetrated deep into the structural pores, capillaries and microcracks of the material. Due to this, the welding of ceramic parts was carried out with sufficient strength. In our case, layers of glass and steel can be rigidly connected using this technology. Steel is a polycrystalline material with many microcracks between the grains among the carbides. Therefore, atoms of copper, or other material according to the technology, penetrate into the microcracks of glass and steel and immobilize the tops of the microcracks of the materials.

Glass is also convenient in that it can be poured into the frame of the reinforcement and thus can be further strengthened. As reinforcing elements, metal wire, polysilicate, polymer, polycarbon compounds, which can have a fairly small thickness, can be used.

In (Bogdanov, 2023; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022), a new approach to solving the problems of impact and nonstationary interaction in the elastoplastic mathematical formulation was developed. In these papers like in non-stationary problems (Bogdanov, 2023; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022), the action of the striker is replaced by a distributed load in the contact area, which changes according to a linear law. The contact area remains constant. The developed elastoplastic formulation makes it possible to solve impact problems when the dynamic change in the boundary of the contact area is considered and based on this the movement of the striker as a solid body with a change in the penetration speed is taken into account. Also, such an elastoplastic formulation makes it possible to consider the hardening of the material in the process of nonstationary and impact interaction.

The solution of problems for composite cylindrical shells (Lokteva et al., 2020), elastic half-space (Igumnov et al., 2013), elastic layer (Kuznetsova et al., 2013), elastic rod (Fedotenkov et al., 2019; Vahterova & Fedotenkov, 2020) were developed using method of the influence functions (Gorshkov & Tarlakovsky, 1985).

In (Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022) dynamic interaction process of plane hard body and two layers reinforced composite material was investigated and the fields of summary plastic deformations and normal stresses arising in the base are calculated using plane strain (PSS) (Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022) and plane stress (PStS) (Bogdanov, 2022) states models. In (Bogdanov, 2022) results are depending on the size of the area of an initial contact between the impactor and the upper surface of the base. In (Bogdanov, 2022) results were calculated depending on the thickness of top metal layer of the composite base. In (Bogdanov, 2022) results were calculated depending on the material of top layer of the composite base. It was investigated composite bases reinforced by steel, titanium and aluminium top layers.

In contrast from the work (Bogdanov, 2018), in this paper, we investigate the impact process of hard body with plane area of its surface on the top of the composite beam which consists first thin metal layer and second main glass layer. In contrast from the works (Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022), the fields of plastic deformations and, stresses were determined relative to the PSS and PStS models in elastic-plastic formulation.

Problem Formulation

Deformations and their increments (Bogdanov, 2023), Odquist parameter $\kappa = \int d\varepsilon_i^p$ (ε_i^p is plastic deformations intensity), stresses are obtained from the numerical solution of the dynamic elastic-plastic interaction problem of infinite composite beam $\{-L/2 \le x \le L/2; 0 \le y \le B; -\infty \le z \le \infty\}$, in case of the problem of PSS, and thin specimen $\{-L/2 \le x \le L/2; 0 \le y \le B\}$, in case of the problem of PStS, in the plane of its cross section in the form of rectangle. It is assumed that the stress-strain state in each cross section of the beam is the same, close to the plane deformation, in case of the problem of PSS, and close to the plane stress, in case of the problem of PStS, and therefore it is necessary to solve the equation for only one section in the form of a rectangle $\Sigma = L \times B$ with two layers: first steel layer $\{-L/2 \le x \le L/2; -\infty \le z \le \infty; B-h \le y \le B\}$ and second glass layer $\{-L/2 \le x \le L/2; -\infty \le z \le \infty; 0 \le y \le B-h\}$ in case of the problem of PSS, and first steel layer $\{-L/2 \le x \le L/2; B-h \le y \le B\}$ and second glass layer $\{-L/2 \le x \le L/2; 0 \le y \le B-h\}$ in case of the problem of PStS contacts absolute hard half-space $\{y \le 0\}$. We assume that the contact between the lower surface of the first metal layer and the upper surface of the second glass layer is ideally rigid.

From above on a body the absolutely rigid drummer is contacting along a segment $\{|x| \le A; y = B\}$. Its action is replaced by an even distributed stress $-P$ in the contact region, which changes over time as a linear function $P = p_{01} + p_{02}t$. Given the symmetry of the deformation process relative to the line x = 0, only the right part of the cross section is considered below (Fig. 1). The calculations use known methods for studying the quasi-static elastic-plastic (Bogdanov, 2023; Mahnenko, 1976; Mahnenko, 2003; Mahnenko, 2009) model, considering the non-stationarity of the load and using numerical integration implemented in the calculation of the dynamic elastic model (Bogdanov, 2023; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022).

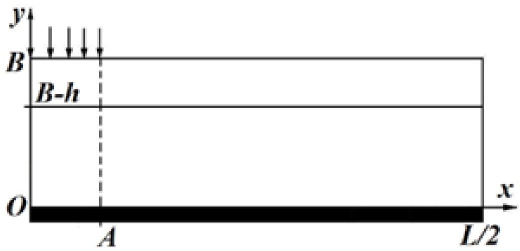

Figure 1: Geometric scheme of the problem

The equations of the plane dynamic theory are considered, for which the components of the displacement vector $\mathbf{u} = (u_x, u_y)$ are related to the components of the strain tensor by Cauchy relations:

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial u_y}{\partial y}, \quad \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right).$$

The equations of motion of the medium have the form:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = \rho \frac{\partial^2 u_x}{\partial t^2}, \quad \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = \rho \frac{\partial^2 u_y}{\partial t^2}, \tag{1}$$

where ρ − material density.

The boundary and initial conditions of the problem have the form:

$$\begin{aligned} & x = 0, \ 0 < y < B: \ u_x = 0, \ \sigma_{xy} = 0, \\ & x = L/2, \ 0 < y < B: \ \sigma_{xx} = 0, \ \sigma_{xy} = 0, \\ & y = 0, \ 0 < x < L/2: \ u_y = 0, \ \sigma_{xy} = 0, \\ & y = B, \ 0 < x < A: \ \sigma_{yy} = -P, \ \sigma_{xy} = 0, \\ & y = B, \ A < x < L/2: \ \sigma_{yy} = 0, \ \sigma_{xy} = 0. \end{aligned} \tag{2}$$

$$u_x\big|_{t=0} = 0, \ u_y\big|_{t=0} = 0, \ u_z\big|_{t=0} = 0, \ \dot{u}_x\big|_{t=0} = 0, \ \dot{u}_y\big|_{t=0} = 0, \ \dot{u}_z\big|_{t=0} = 0. \tag{3}$$

The determinant relations of the mechanical model are based on the theory of non-isothermal plastic flow of the medium with hardening under the condition of Huber-Mises fluidity. The effects of creep and thermal expansion are neglected. Then, considering the components of the strain tensor by the sum of its elastic and plastic components (Kachanov, 1969; Collection: Theory of plasticity IL, 1948), we obtain expression for them:

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p},\ d\varepsilon_{ij}^{p} = s_{ij} d\lambda,\ \varepsilon_{ij}^{e} = \frac{1}{2G} s_{ij} + K\sigma + \varphi. \qquad (4)$$

here $s_{ij} = \sigma_{ij} - \delta_{ij}\sigma$ – stress tensor deviator; δ_{ij} – Kronecker symbol; E – modulus of elasticity (Young's modulus); G – shear modulus; $K_1 = (1-2\nu)/(3E)$, $K = 3K_1$ –volumetric compression modulus, which binds in the ratio $\varepsilon = K\sigma + \phi$ volumetric expansion 3ε (thermal expansion $\phi = 0$); $\sigma = (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3$ – mean stress in the case of PSS and $\sigma = (\sigma_{xx} + \sigma_{yy})/3$ – mean stress in the case of PStS; $d\lambda$ – some scalar function ((Mahnenko, 1976), which is determined by the shape of the load surface and we assume that this scalar function is quadratic function of the stress deviator s_{ij} (Mahnenko, 1976; Kachanov, 1969; Collection: Theory of plasticity IL, 1948).

$$d\lambda = \begin{cases} 0 \quad (f \equiv \sigma_i^2 - \sigma_S^2(T) < 0) \\ \dfrac{3d\varepsilon_i^p}{2\sigma_i} \quad (f = 0, df = 0) \\ (f > 0 - \text{inadmissible}) \end{cases}, \qquad (5)$$

$$d\varepsilon_i^p = \frac{\sqrt{2}}{3}\left(\left(d\varepsilon_{xx}^p - d\varepsilon_{yy}^p\right)^2 + \left(d\varepsilon_{xx}^p - d\varepsilon_{zz}^p\right)^2 + \left(d\varepsilon_{yy}^p - d\varepsilon_{zz}^p\right)^2 + 6\left(d\varepsilon_{xy}^p\right)^2\right)^{1/2},$$

$$\sigma_i = \frac{1}{\sqrt{2}}\left(\left(\sigma_{xx} - \sigma_{yy}\right)^2 + \left(\sigma_{xx} - \sigma_{zz}\right)^2 + \left(\sigma_{yy} - \sigma_{zz}\right)^2 + 6\sigma_{xy}^2\right)^{1/2}.$$

In case of PStS stress intensity would be as follow:

$$\sigma_i = \frac{1}{\sqrt{2}}\left(\left(\sigma_{xx} - \sigma_{yy}\right)^2 + \left(\sigma_{xx}\right)^2 + \left(\sigma_{yy}\right)^2 + 6\sigma_{xy}^2\right)^{1/2}.$$

The material is strengthened with a hardening factor η^* (Bogdanov, 2023; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022; Mahnenko, 1976):

$$\sigma_S(T) = \sigma_{02}(T_0)\left(1 + \frac{\kappa(T)}{\varepsilon_0}\right)^{\eta^*},\ \varepsilon_0 = \frac{\sigma_{02}(T_0)}{E}, \qquad (6)$$

where T – temperature; k – Odquist parameter, $T_0 = 20^\circ C$, η^* – hardening coefficient; $\sigma_S(T)$ – yield strength after hardening of the material at temperature T.

Rewrite (4) in expanded form:

$$d\varepsilon_{xx} = d\left(\frac{\sigma_{xx} - \sigma}{2G} + K\sigma\right) + (\sigma_{xx} - \sigma)d\lambda,\ d\varepsilon_{yy} = d\left(\frac{\sigma_{yy} - \sigma}{2G} + K\sigma\right) + (\sigma_{yy} - \sigma)d\lambda,$$
$$d\varepsilon_{zz} = d\left(\frac{\sigma_{zz} - \sigma}{2G} + K\sigma\right) + (\sigma_{zz} - \sigma)d\lambda,\ d\varepsilon_{xy} = d\left(\frac{\sigma_{xy}}{2G}\right) + \sigma_{xy} d\lambda, \qquad (7)$$

In contrast to the traditional plane deformation, when $\Delta\varepsilon_{zz}(x, y) = \text{const}$, for a refined description of the deformation of the specimen, taking into account the possible increase in longitudinal elongation $\Delta\varepsilon_{zz}$, we present in its form (Bogdanov, 2023; Boli&Waner, 1964):

$$\Delta\varepsilon_{zz}(x, y) = \Delta\varepsilon_{zz}^0 + \Delta\chi_x x + \Delta\chi_y y, \qquad (8)$$

Where unknown $\Delta\chi_x$ and $\Delta\chi_y$ describe the bending of the prismatic body (which simulates the plane strain state in the solid mechanics) in the *Ozx* and *Ozy* planes, respectively, and $\Delta\varepsilon_{zz}^0$ – the increments according to the detected deformation bending along the fibres $x = y = 0$

In case of PStS it is necessary to exclude ε_{zz} from (7).

Solution Algorithm

Let the nonstationary interaction (Bogdanov, 2023) occurs in a time interval $t \in [0, t_*]$. Then for every moment of time t:

$$\varepsilon_{xx}^e = \frac{\sigma_{xx} - \sigma}{2G} + K\sigma,\ \varepsilon_{yy}^e = \frac{\sigma_{yy} - \sigma}{2G} + K\sigma,\ \varepsilon_{zz}^e = \frac{\sigma_{zz} - \sigma}{2G} + K\sigma,\ \varepsilon_{xy}^e = \frac{\sigma_{xy}}{2G}, \qquad (9)$$

$$\frac{d\varepsilon_{xx}^p}{dt} = (\sigma_{xx} - \sigma)\frac{d\lambda}{dt},\ \frac{d\varepsilon_{xy}^p}{dt} = \sigma_{xy}\frac{d\lambda}{dt},\ \frac{d\varepsilon_{yy}^p}{dt} = \left(\sigma_{yy} - \sigma\right)\frac{d\lambda}{dt},\ \frac{d\varepsilon_{zz}^p}{dt} = (\sigma_{zz} - \sigma)\frac{d\lambda}{dt}.$$

In case of PStS in (9) $\varepsilon_{zz}^e = -\dfrac{\nu}{1-\nu}(\varepsilon_{xx}^e + \varepsilon_{yy}^e),\ \varepsilon_{zz}^p = -\varepsilon_{xx}^p - \varepsilon_{yy}^p$.

For numerical integration over time, Gregory's quadrature formula (Bogdanov, 2023; Hemming, 1972) of order m_1 = 3 with coefficients D_n was used. After discretisation in time with nodes $t_k = k\Delta t \in [0, t_*]$ $(k = 0, K)$ for each value k, we write down the corresponding node values of deformation increments in case of PSS:

$$\Delta\varepsilon_{xx,k} = B_1\sigma_{xx,k} + B_2\sigma_{yy,k} - \beta_{xx},\ \Delta\varepsilon_{yy,k} = B_2\sigma_{xx,k} + B_1\sigma_{yy,k} - \beta_{yy},$$
$$\Delta\varepsilon_{zz,k} = \alpha_1\sigma_{zz,k} + \alpha_2(\sigma_{xx,k} - \sigma_{yy,k}) - b_{zz},\ \Delta\varepsilon_{xy,k} = B_3\sigma_{xy,k} - b_{xy},$$
$$\beta_{xx} = b_{xx} - \alpha_2(b_{zz} + \Delta\varepsilon_{zz})/\alpha_1,\ \beta_{yy} = b_{yy} - \alpha_2(b_{zz} + \Delta\varepsilon_{zz})/\alpha_1,\ \beta_{zz} = -(b_{zz} + \Delta\varepsilon_{zz})/\alpha_1,$$
$$B_1 = \frac{\alpha_1^2 - \alpha_2^2}{\alpha_1},\ B_2 = \frac{\alpha_2(\alpha_1 - \alpha_2)}{\alpha_1},\ B_3 = \frac{1}{2G} + D_0\Delta\lambda_k,$$
$$\alpha_1 = \frac{1}{3}\left(K + \frac{1}{G} + 2D_0\Delta\lambda_k\right),\ \alpha_2 = \frac{1}{3}\left(K - \frac{1}{2G} - D_0\Delta\lambda_k\right),$$
$$b_{ij} = \frac{1}{2G}\sigma_{ij,k-1} + \delta_{ij}\left(K - \frac{1}{2G}\right)\sigma_{k-1} - \sum_{n=1}^{m_1} D_n\left(\sigma_{ij,k-n} - \delta_{ij}\sigma_{k-n}\right)\Delta\lambda_{k-n}\ \ (i, j = x, y, z). \qquad (10)$$

and in case of PStS (Bogdanov, 2023):

$$\Delta\varepsilon_{xx,k} = \tilde{B}_1\sigma_{xx,k} + \tilde{B}_2\sigma_{yy,k} - \tilde{b}_{xx},\ \Delta\varepsilon_{yy,k} = \tilde{B}_2\sigma_{xx,k} + \tilde{B}_1\sigma_{yy,k} - \tilde{b}_{yy},$$
$$\Delta\varepsilon_{xy,k} = \tilde{B}_3\sigma_{xy,k} - \tilde{b}_{xy},\ \Delta\varepsilon_{zz,k} = \tilde{B}_4(\sigma_{xx,k} + \sigma_{yy,k}) - \Delta\varepsilon_{xx,k} - \Delta\varepsilon_{yy,k} - \tilde{b}_{zz},$$
$$\tilde{B}_1 = \frac{1}{3}\left(K + \frac{1}{G} + 2D_0\Delta\lambda_k\right),\ \tilde{B}_2 = \frac{1}{3}\left(K - \frac{1}{2G} - D_0\Delta\lambda_k\right),$$
$$\tilde{B}_3 = \frac{1}{2G} + D_0\Delta\lambda_k,\ \tilde{B}_4 = \frac{1-2\nu}{3(1-\nu)}\left(2K + \frac{1}{2G}\right),\ \tilde{b}_{zz} = \tilde{B}_4(\sigma_{xx,k-1} + \sigma_{yy,k-1}),$$
$$\tilde{b}_{ij} = \frac{1}{2G}\sigma_{ij,k-1} + \delta_{ij}\left(K - \frac{1}{2G}\right)\sigma_{k-1} - \sum_{n=1}^{m_1-1} D_n\left(\sigma_{ij,k-n} - \delta_{ij}\sigma_{k-n}\right)\Delta\lambda_{k-n}$$
$$(i, j \square\ x, y). \qquad (11)$$

The solution of the system (10), in case of PSS, gives expressions for the components of the stress tensor at each step (Bogdanov, 2023):

$$\sigma_{xx,k} = A_1\Delta\varepsilon_{xx,k} + A_2\Delta\varepsilon_{yy,k} + Y_{xx},\ \sigma_{yy,k} = A_2\Delta\varepsilon_{xx,k} + A_1\Delta\varepsilon_{yy,k} + Y_{yy},$$
$$\sigma_{zz,k} = -\alpha_2(\sigma_{xx,k} + \sigma_{yy,k})/\alpha_1 - \beta_{zz},\ \sigma_{xy,k} = A_3\Delta\varepsilon_{xy,k} + Y_{xy},$$
$$Y_{xx} = A_1\beta_{xx} + A_2\beta_{yy},\ Y_{yy} = A_2\beta_{xx} + A_1\beta_{yy},\ Y_{xy} = A_3 b_{xy},\ A_3 = 1/B_3,$$
$$A_1 = B_1\Big/\left(B_1^2 - B_2^2\right),\ A_2 = -B_2\Big/\left(B_1^2 - B_2^2\right). \qquad (12)$$

The solution of the system (11), in case of PStS, gives expressions for the components of the stress tensor at each step as follow (Bogdanov, 2023):

$$\sigma_{xx,k} = \tilde{A}_1\Delta\varepsilon_{xx,k} + \tilde{A}_2\Delta\varepsilon_{yy,k} + \tilde{Y}_{xx},\ \tilde{Y}_{xx} = \tilde{A}_1\tilde{b}_{xx} + \tilde{A}_2\tilde{b}_{yy},\ \tilde{A}_1 = \tilde{B}_1\Big/\left(\tilde{B}_1^2 - \tilde{B}_2^2\right),$$
$$\sigma_{yy,k} = \tilde{A}_2\Delta\varepsilon_{xx,k} + \tilde{A}_1\Delta\varepsilon_{yy,k} + \tilde{Y}_{yy},\ \tilde{Y}_{yy} = \tilde{A}_2\tilde{b}_{xx} + \tilde{A}_1\tilde{b}_{yy},\ \tilde{A}_2 = -\tilde{B}_2\Big/\left(\tilde{B}_1^2 - \tilde{B}_2^2\right),$$
$$\sigma_{xy,k} = \tilde{A}_3\Delta\varepsilon_{xy,k} + \tilde{Y}_{xy},\ \tilde{Y}_{xy} = \tilde{A}_3\tilde{b}_{xy},\ \tilde{A}_3 = 1/\tilde{B}_3. \quad (13)$$

Function $\psi = 1/(2G) + \Delta\lambda$, which is characterizing the yield condition, taking into account (8), (9) and in case of PSS (12) or in case of PStS (13) is:

$$\psi = \begin{cases} \dfrac{1}{2G} & (f < 0) \\ \dfrac{1}{2G} + \dfrac{3\Delta\varepsilon_i^p}{2\sigma_i} & (f = 0, df = 0), \\ (f > 0 - \text{inadmissible}) & \end{cases} \quad (14)$$

$$\Delta\varepsilon_i^p = \frac{\sqrt{2}}{3}\left(\left(\Delta\varepsilon_{xx}^p - \Delta\varepsilon_{yy}^p\right)^2 + \left(\Delta\varepsilon_{xx}^p - \Delta\varepsilon_{zz}^p\right)^2 + \left(\Delta\varepsilon_{yy}^p - \Delta\varepsilon_{zz}^p\right)^2 + 6\left(\Delta\varepsilon_{xy}^p\right)^2\right)^{1/2},$$
$$\Delta\varepsilon_{xx}^p = \Delta\varepsilon_{xx} - \Delta\varepsilon_{xx}^e,\ \Delta\varepsilon_{yy}^p = \Delta\varepsilon_{yy} - \Delta\varepsilon_{yy}^e,\ \Delta\varepsilon_{xy}^p = \Delta\varepsilon_{xy} - \Delta\varepsilon_{xy}^e,\ \Delta\varepsilon_{zz}^p = \Delta\varepsilon_{zz} - \Delta\varepsilon_{zz}^e,$$
$$\Delta\varepsilon_{xx}^e = \frac{1}{2G}\sigma_{xx} + \left(K - \frac{1}{2G}\right)\sigma,\ \Delta\varepsilon_{yy}^e = \frac{1}{2G}\sigma_{yy} + \left(K - \frac{1}{2G}\right)\sigma,$$
$$\Delta\varepsilon_{zz}^e = \frac{1}{2G}\sigma_{zz} + \left(K - \frac{1}{2G}\right)\sigma,\ \Delta\varepsilon_{xy}^e = \frac{1}{2G}\sigma_{xy},\ \sigma = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3}.$$

In case of PStS $\Delta\varepsilon_{zz}^p = -\Delta\varepsilon_{xx}^p - \Delta\varepsilon_{yy}^p,\ \Delta\varepsilon_{xy}^e = \dfrac{1}{2G}\sigma_{xy},\ \Delta\varepsilon_{zz}^e = 0,\ \sigma = \dfrac{\sigma_{xx} + \sigma_{yy}}{3}.$

Considering when calculating the value $\Delta\varepsilon_{zz}^p$, we found that its impact is so small that without reducing the accuracy of calculations can be considered $\Delta\varepsilon_{zz}^p = 0$.
To take into account (Bogdanov, 2023) the physical nonlinearity contained in conditions (14), the method of successive approximations is used, which makes it possible to reduce a nonlinear problem to a sequence of linear problems (Bogdanov, 2023; Mahnenko, 1976; Mahnenko, 2003;Mahnenko et al., 2009):

$$\psi^{(n+1)} = \left\{\psi^{(n)}p + \frac{1-p}{2G},\ \text{if } \sigma_{iS} < -Q;\ \psi^{(n)},\ \text{if } -Q < \sigma_{iS} < Q;\ \psi^{(n)}\frac{\sigma_i^{(n)}}{\sigma_S(T)},\ \text{if } \sigma_{iS} > Q\right\},$$
$$\sigma_{iS} = \sigma_i^{(n)} - \sigma_S(T), \quad (15)$$

where Q – the value of the largest deviation of the stress intensity $\sigma_i^{(n)}$ in step n from the strengthened yield strength; *n* – is the approximation number.

In case of PSS, unknown (Boli & Waner, 1964) $\Delta\chi_x$, $\Delta\chi_y$ and $\Delta\varepsilon_{zz}^0$ in (8) are determined from the conditions of equilibrium of even with respect to *x* normal stresses σ_{zz}.

$$\iint_\Sigma \sigma_{zz}(x,y)\rho dxdy = M_\rho,\ (\rho = 1, x, y), \quad (16)$$

When $M_1 = M_x = M_y = 0$; where M_1 – projection on the axis *Oz* of the main vector of contact stresses, and M_x, M_y – corresponding projections of the main moment of the forces acting on the resistance (no torsion, as noted).
Given the symmetry of the problem and $\sigma_{zz}(x,y) = \sigma_{zz}(-x,y)$ this equation in case of $\rho = x$ is satisfied automatically.

If we substitute (8) and (12) in (16), taking into account the symmetry of the integration domain with respect to *x* and the even of functions $\sigma_{xx,k}, \sigma_{yy,k}, {}_{zz}$, we have $\Delta\chi_x = 0$. A system of linear algebraic equations is obtained for the calculation of $\Delta\varepsilon_{zz}^0,\ \Delta\chi_y$:

$$\Delta\varepsilon_{zz}^0 L_{\rho 1} + \Delta\chi_y L_{\rho y} = \bar{M}_\rho,\ (\rho = 1, y),$$
$$\bar{M}_\rho = \iint_\Sigma \frac{\alpha_2\left(\sigma_{xx} + \sigma_{yy}\right) - b_{zz}}{\alpha_1}\rho dxdy,\ L_{\rho r} = \iint_\Sigma \frac{\rho dxdy}{\alpha_1},\ (r, \rho = 1, x, y). \quad (17)$$

The stresses and strains used above were determined for each unit cell from the numerical solution at each point in time $t_k = k\Delta t$.

Numerical Solution

For both problems the explicit scheme of the finite difference method was used with a variable partitioning step along the axes *Ox* (*M* elements) and *Oy* (*N* elements). The step between the split points was the smallest in the area of the layers contact and at the boundaries of the computational domain. Since the interaction process is fleeting, this did not affect the accuracy in the first thin layer, areas near the boundaries, and the adequacy of the contact interaction modelling.

The use of finite differences (Hemming, 1972) with variable partition step for wave equations is justified in (Zukina, 2004), and the accuracy of calculations with an error of no more than $O\left((\Delta x)^2 + (\Delta y)^2 + (\Delta t)^2\right)$ where Δx, Δy and Δt – increments of variables: spatial *x* and *y* and time *t*. A low rate of change in the size of the steps of the partition mesh was ensured. The time step was constant.

The resolving system of linear algebraic equations with a banded symmetric matrix was solved by the Gauss method according to the Cholesky scheme.

In (Weisbrod & Rittel, 2000), during experiments, compact samples were destroyed in 21 – 23 ms. The process of destruction of compact specimens from a material of size and with contact loading as in (Weisbrod & Rittel, 2000) was modelled in a dynamic elastoplastic formulation as plane strain state, considering the unloading of the material and the growth of a crack according to the local criterion of brittle fracture. The samples were destroyed in 23 *ms*. This confirms the correctness and adequacy of the developed formulation and model.

Figs. 2 – 29 show the results of calculations of two layers specimens with a hardening factor of the material $\eta^* = 0{,}05$. The first high layer has made from hard steel. The second main low layer has made from quartz glass. Contact between two layers is an ideal. Calculations were made at the following parameter values: temperature $T = 50$ °C; $L = 60$ *mm*; $B = 10mm$; $h = 0.5mm$; $\Delta t = 3.21 \cdot 10^{-8}$ *s*; $P_{01} = 8MPa$; $P_{02} = 10MPa$; $M = 62$; $N = 100$. The smallest splitting step was 0,005 *mm*, and the largest 2,6 *mm* ($\Delta x_{\min} = 0{,}005\ mm$; $\Delta y_{\min} = 0{,}01\ mm$ (only the first layer); $\Delta x_{\max} = 2{,}6\ mm$; $\Delta y_{\max} = 0{,}65\ mm$).

Fig. 2 shows plots of the Odquist parameter k in the cell of the first layer, which is located in the centre of the specimen with first layer $h = 0.3mm$ thick at a depth of 0.25 *mm*. Solid, dotted, and solid with a circle lines correspond to cases where the size of [2] the contact zone was equal $a = 2A = a_1 = 0.3\ mm$, $a = a_2 = 0.5\ mm$ and $a = a_3 = 0.7\ mm$, respectively.

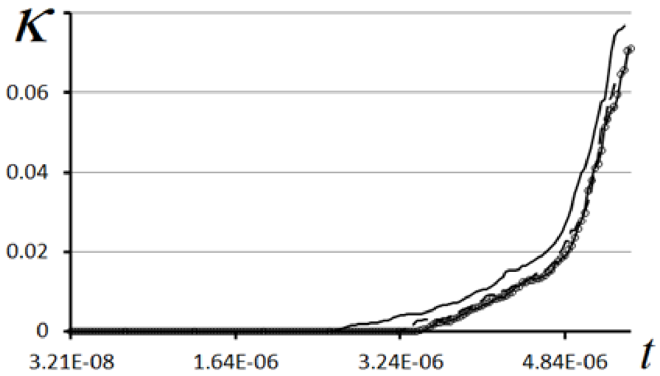

Figure 2: Odquist parameter k when $t = t_1$

Further Figs. represent results for contact zone size $a = a_2$. Figs. 3, 6, 9, 12, 15, 18; 4, 7, 10, 13, 16, 19; 5, 8, 11, 14, 17, 20 show the fields of the Odquist parameter K, normal stresses σ_{xx} and σ_{yy} at times $t_1 = 2.57 \cdot 10^{-6}\ s$, $t_2 = 3.82 \cdot 10^{-6}\ s$ and $t_3 = 4.33 \cdot 10^{-6}\ s$, respectively.

From Figs. 3 – 8 it can be seen that in the area under the contact zone the plastic deformations are bigger and quicker in the case of PStS and at the end of the process of non-stationary interaction, when the moment of time t_3 they are of the higher degree.

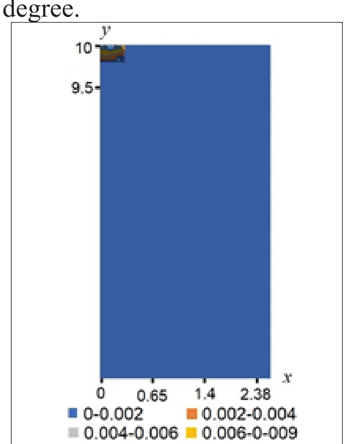

Fig. 3. PStS. Odquist parameter K when $t = t_1$

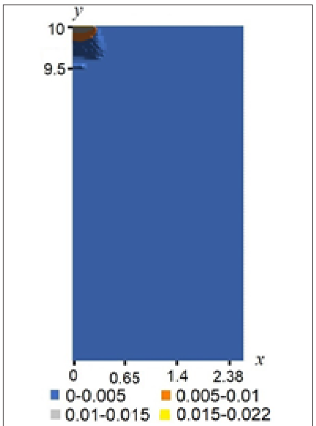

Fig. 4. PStS. Odquist parameter K when $t = t_2$

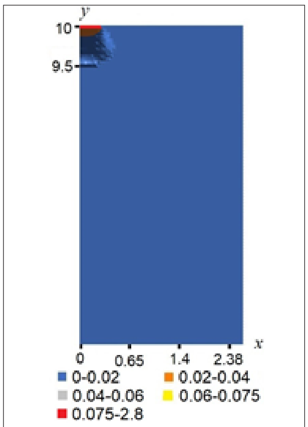

Fig. 5. PStS. Odquist parameter K when $t = t_3$

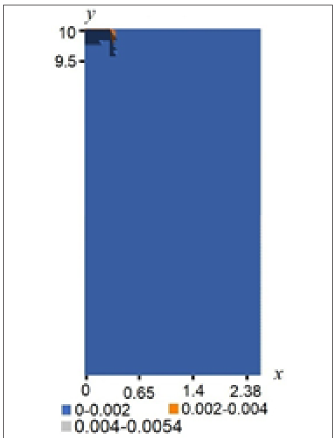

Fig. 6. PSS. Odquist parameter K when $t = t_1$

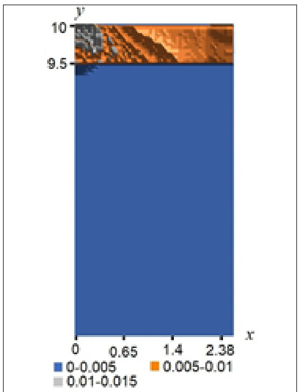

Fig. 7. PSS. Odquist parameter K when $t = t_2$

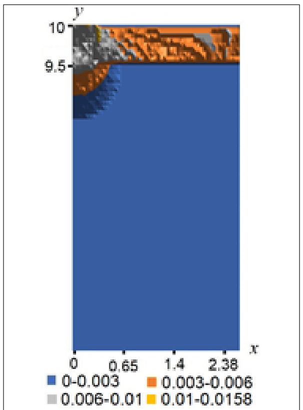

Fig. 8. PSS. Odquist parameter K when $t = t_3$

Figs. 9 – 20 show that the highest stresses occur in the upper layer of the metal and the process of accumulation of plastic deformations is more intense there. These Figs. show areas where the normal stresses in layers are tensile. This is due to the fact that compressive stresses arise in the upper layer quickly and the contact between the layers and the contact of the lower boundary of the lower layer with an absolutely rigid base are ideally rigid.

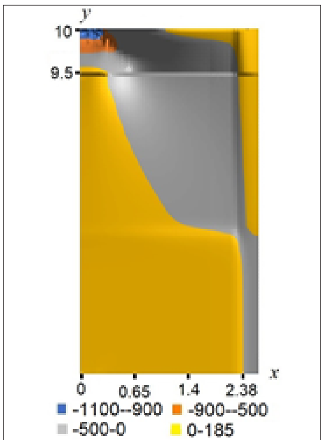

Fig. 9. PStS. Stress σ_{xx} when $t = t_1$

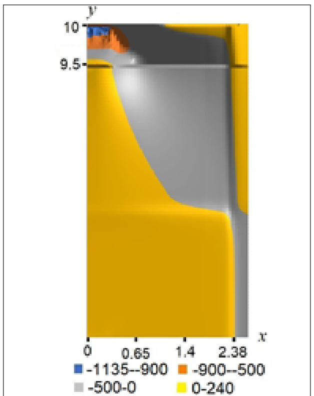

Fig. 10. PStS. Stress σ_{xx} when $t = t_2$

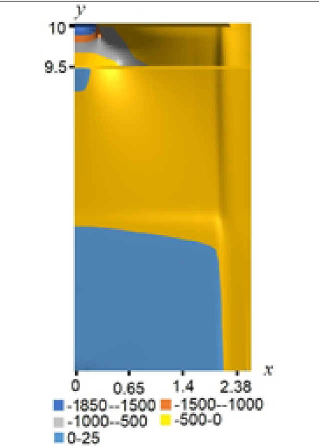

Fig. 12. PSS. Stress σ_{xx} when $t = t_1$

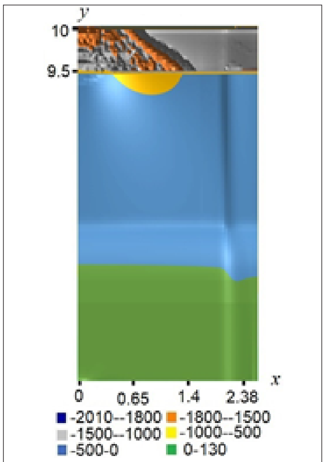

Fig. 13. PSS. Stress σ_{xx} when $t = t_2$

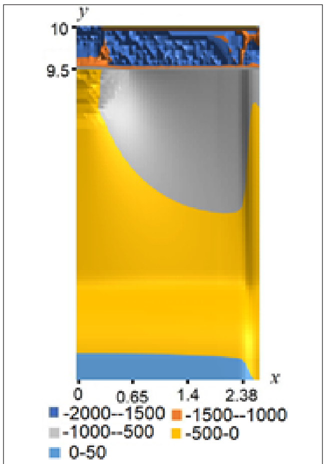

Fig. 14. PSS. Stress σ_{xx} when $t = t_3$

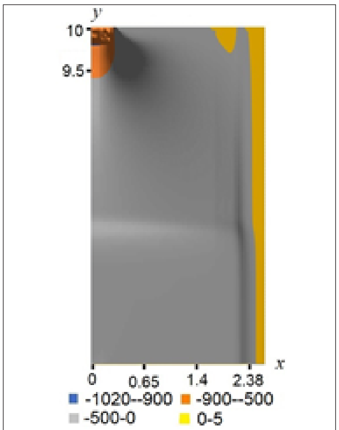

Fig. 15. PStS. Stress σ_{yy} when $t = t_1$

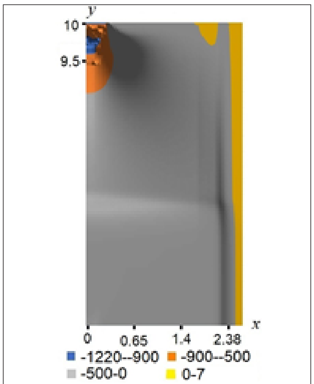

Fig. 16. PStS. Stress σ_{yy} when $t = t_2$

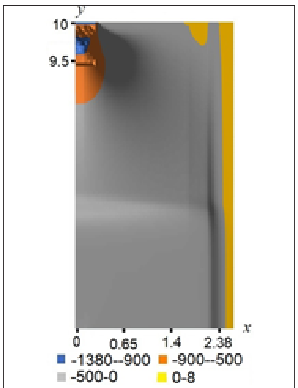

Fig. 17. PStS. Stress σ_{yy} when $t = t_3$

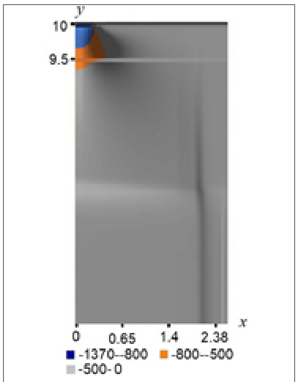

Fig. 18. PSS. Stress σ_{yy} when $t = t_1$

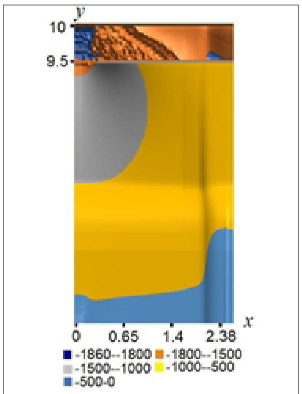

Fig. 19. PSS. Stress σ_{yy} when $t = t_2$

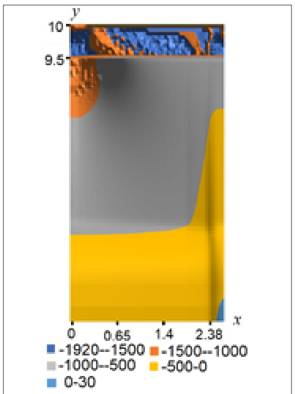

Fig. 20. PSS. Stress σ_{yy} when $t = t_3$

The summary plastic deformations at time t_1 in the case of PStS are 40% greater than in the case of PSS (Bogdanov, 2022) and the area where these plastic deformations occur is slightly larger. At times t_2 and t_3 , the area of plastic deformations in the case of PStS is located under the contact zone, and the summary plastic deformations are greater in magnitude than in the case of PSS (Bogdanov, 2022) by 32% and 99% at times t_2 and t_3 respectively. In the case of PStS, the large in absolute value normal stresses σ_{xx} and σ_{yy} arise in the area under the contact zone. Moreover, the largest values of normal stresses in the case of PStS are less in absolute value than the values in the case of PSS (Bogdanov, 2022) at the times t_1, t_2 and t_3, respectively, by 41%, 44% and 43%. The largest absolute values of normal stresses σ_{yy} in the case of PStS are less than the corresponding values in the case of PSS at the same time points by 25%, 34% and 28%, respectively.

The PStS simulates the process of impact on a narrow strip of a two-layer base. In the case of PStS, plastic deformations grow much faster than in the case of PSS.

Conclusions

The developed methodology of solving dynamic contact problems in an elastic-plastic dynamic mathematical formulation makes it possible to model the processes of impact, shock and non-stationary contact interaction with the elastic composite base adequately. In this work, the process of impact on a two-layers base, consisting of an upper thin layer of metal and a lower main layer of glass, is adequately modelled and investigated. The fields of summary plastic deformations and normal stresses arising in the base are calculated and compared to the corresponding values from the cor-responding problems of plane strain and stress states. The upper metal layer of the composite two-layer base takes on the main load. The results obtained make it possible to design the narrow strips of new composite reinforced armed materials. Such a two-layer reinforced composite material can be used as a wide range of needs of modern industry.

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