Research Article ISSN 2832-9384

Journal of Materials and Polymer Science

Plane Strain State of Four-Layers Composite Reinforced Body in Dynamic Elastic-Plastic Formulation

Vladislav Bogdanov

Progressive Research Solutions Pty. Ltd.

***Correspondence author**
Vladislav Bogdanov,
Progressive Research Solutions Pty. Ltd.
28/2 Buller Rd, Artarmon, Sydney,
Australia 2064.

Submitted : 17 Apr 2023 ; Published : 6 Jun 2023

Citation : Bogdanov, V (2023). Plane Strain State of Four-Layers Composite Reinforced Body in Dynamic Elastic-Plastic Formulation. *J mate poly sci, 3*(2): 1-7.

Abstract

The design of composite and reinforced or armed materials is a requirement of the modern level of production and life. In previous works (Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022), the plane problems of non-stationary interaction of a bullet-type impactor with the upper surface of a composite reinforced two-layers base, which consists of an upper thin layer of steel and a lower main layer of glass, was investigated. It is of interest to study the question of how a composite material composed of several two-layers composite bases laid on top of each other and rigidly bonded to each other will behave. In this work, we study the non-stationary interaction of a striker and a four-layers composite material reinforced with two thin steel layers. The four-layers base is obtained from two identical two-layers bases rigidly linked to each other. Such a four-layer material along its lower surface is rigidly linked to an absolutely hard half-space. The main layers of the material consist of glass. The use of glass in composites is promising due to the fact that glass is a durable, cheap, widespread material that does not corrode, and its strength properties do not degrade as a result of aging, creep like other materials, especially metals. The problem of glass brittleness is overcome by hard contact between the layers. In this case, the tops of micro cracks, micro pores on the surfaces of glass and steel are immobilized and do not propagate into the layers. An absolutely solid impactor acts from above in the centre on a small area of initial contact. The problem of a plane strain state of a beam made from the composite reinforced four layers material is being solved. A technique for solving dynamic contact problems in a dynamic elastic-plastic mathematical formulation is used. To consider the physical nonlinearity of the deformation process, the method of successive approximations is used, which makes it possible to reduce the nonlinear problem to a solution of the sequences of linear problems.

Keywords: Plane, strain, stress, state, impact, composite, armed, reinforced, material, elastic-plastic, deformation.

Introduction

In (Bogdanov, 2023; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022), a new approach to solving the problems of impact and nonstationary interaction in the elastoplastic mathematical formulation was developed. In these papers like in non-stationary problems (Bogdanov, 2023; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022), the action of the striker is replaced by a distributed load in the contact area, which changes according to a linear law. The contact area remains constant. Such an elastoplastic formulation makes it possible to consider the hardening of the material in the process of nonstationary and impact interaction.

The solution of problems for composite cylindrical shells (Lokteva et al., 2020), elastic half-space (Igumnov et al., 2013), elastic layer (Kuznetsova et al., 2013), elastic rod (Fedotenkov et al., 2019; Vahterova & Fedotenkov, 2020) were developed using method of the influence functions (Gorshkov & Tarlakovsky, 1985).

In contrast from the work (Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2018), in this paper, we investigate the impact process of hard body with plane area of its surface on the top of the composite beam which consists of first and third thin metal layers and second and fourth main glass layers (Fig. 1). The fields of parameter Odquist and stresses were determined relative to the number of layers.

Problem Formulation

Deformations and their increments (Bogdanov, 2012; Mahnenko, 1976), Odquist parameter $\kappa = \int d\varepsilon_i^p$ (ε_i^p is plastic deformations intensity), effective and principal stresses are obtained from the numerical solution of the dynamic elastic-plastic interaction problem of infinite composite beam $\{-L/2 \le x \le L/2; 0 \le y \le B; -\infty \le z \le \infty\}$ in the plane of its cross section in the form of rectangle. It is assumed that the stress-strain state in each cross section of the cylinder is the same, close to the plane deformation, and therefore it is necessary to solve the equation for only one section in the form of a rectangle $\Sigma = L$ x B with four layers: first steel layer $\{-L/2 \le x \le L/2; -\infty \le z \le \infty;, \ B - h_1 \le y \le B\}$,second glass layer $\{-L/2 \le x \le L/2;, -\infty \le z \le \infty; B/2 \le y \le B - h_1\}$ third steel layer $\{-L/2 \le x \le L/2; -\infty \le z \le \infty; \ B/2 - h_2 \le y \le B/2\}$ and fourth glass layer $\{-L/2 \le x \le L/2; \quad -\infty \le z \le \infty; 0 \le y \le B/2 - h_2\}$ contacts absolute hard half-space$\{ \ y \le 0 \ \}$, here h_1, h_2 are thicknesses of first and third steel layers as at Fig. 1. We assume that the contacts between layers are ideally rigid.

From above on a body the absolutely rigid drummer contacting along a segment $\{|x| \le A; \ y = B\}$. Its action is replaced by an even distributed stress *-P* in the contact region, which changes over time as a linear function $P = p_{01} + P_{02}t$. Given the symmetry of the deformation process relative to the line
$x = 0$, only the right part of the cross section is considered below (Fig. 1). The calculations use known methods for studying the quasi-static elastic-plastic (Bogdanov, 2012; Mahnenko, 1976; Mahnenko, 2003; Mahnenko, et al., 2009) model and the dynamic elastic-plastic model (Bogdanov, 2023), considering the non-stationarity of the load and using numerical integration implemented in the calculation of the dynamic elastic model (Bogdanov, 2023; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022).

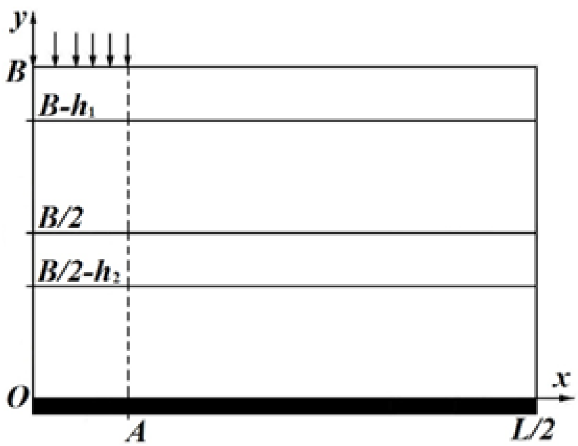

Figure 1: Geometric scheme of the problem

The equations of the plane dynamic theory are considered, for which the components of the displacement vector u = (u_x, u_y) are related to the components of the strain tensor by Cauchy relations:

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial u_y}{\partial y}, \quad \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right).$$

The equations of motion of the medium have the form:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = \rho \frac{\partial^2 u_x}{\partial t^2}, \quad \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = \rho \frac{\partial^2 u_y}{\partial t^2}, \qquad (1)$$

where ρ − material density.

The boundary and initial conditions of the problem have the form:

$$\begin{aligned} &x = 0, \ 0 < y < B: \ u_x = 0, \ \sigma_{xy} = 0, \\ &x = L/2, \ 0 < y < B: \ \sigma_{xx} = 0, \ \sigma_{xy} = 0, \\ &y = 0, \ 0 < x < L/2: \ u_y = 0, \ \sigma_{xy} = 0, \\ &y = B, \ 0 < x < A: \ \sigma_{yy} = -P, \ \sigma_{xy} = 0, \\ &y = B, \ A < x < L/2: \ \sigma_{yy} = 0, \ \sigma_{xy} = 0. \end{aligned} \qquad (2)$$

$$u_x\big|_{t=0} = 0, \ u_y\big|_{t=0} = 0, \ u_z\big|_{t=0} = 0, \ \dot{u}_x\big|_{t=0} = 0, \ \dot{u}_y\big|_{t=0} = 0, \ \dot{u}_z\big|_{t=0} = 0. \qquad (3)$$

The determinant relations of the mechanical model are based on the theory of non-isothermal plastic flow of the medium with hardening under the condition of Huber-Mises fluidity. The effects of creep and thermal expansion are neglected. Then, considering the components of the strain tensor by the sum of its elastic and plastic components (Kachanov, 1969; Collection: Theory of plasticity IL, 1948), we obtain expression for them:

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p, \ d\varepsilon_{ij}^p = s_{ij} d\lambda, \ \varepsilon_{ij}^e = \frac{1}{2G} s_{ij} + K\sigma + \varphi. \qquad (4)$$

here $s_{ij} = \sigma_{ij} - \delta_{ij}\sigma$ – stress tensor deviator; δ_{ij} – Kronecker symbol; E – modulus of elasticity (Young's modulus);
G – shear modulus; $K_1 = (1\text{-}2\nu)/(3E)$, $K = 3K_1$– volumetric compression modulus, which binds in the ratio $\varepsilon = K\sigma + \phi$ volumetric expansion (thermal expansion $\phi = 0$); $\sigma = (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3$ – mean stress; $d\lambda$ – some scalar function (Bogdanov, 2023; Mahnenko, 1976), which is determined by the shape of the load surface and we assume that this scalar function is quadratic function of the stress deviator S_{ij} (Kachanov, 1969; Collection: Theory of plasticity IL, 1948).

$$d\lambda = \begin{cases} 0 \quad (f \equiv \sigma_i^2 - \sigma_S^2(T) < 0) \\ \dfrac{3d\varepsilon_i^p}{2\sigma_i} \quad (f = 0, df = 0) \\ (f > 0 - \text{inadmissible}) \end{cases}, \qquad (5)$$

$$d\varepsilon_i^p = \frac{\sqrt{2}}{3}\left(\left(d\varepsilon_{xx}^p - d\varepsilon_{yy}^p\right)^2 + \left(d\varepsilon_{xx}^p - d\varepsilon_{zz}^p\right)^2 + \left(d\varepsilon_{yy}^p - d\varepsilon_{zz}^p\right)^2 + 6\left(d\varepsilon_{xy}^p\right)^2\right)^{1/2},$$

$$\sigma_i = \frac{1}{\sqrt{2}}\left(\left(\sigma_{xx} - \sigma_{yy}\right)^2 + \left(\sigma_{xx} - \sigma_{zz}\right)^2 + \left(\sigma_{yy} - \sigma_{zz}\right)^2 + 6\sigma_{xy}^2\right)^{1/2}.$$

The material is strengthened with a hardening factor η^* (Bogdanov, 2023; Mahnenko, 1976):

$$\sigma_S(T) = \sigma_{02}(T_0)\left(1 + \frac{\kappa(T)}{\varepsilon_0}\right)^{\eta^*}, \ \varepsilon_0 = \frac{\sigma_{02}(T_0)}{E}, \qquad (6)$$

where T – temperature; k – Odquist parameter, $T_0 = 20^{\circ}$C,
η^* – hardening coefficient; $\sigma_s(T)$ – yield strength after hardening of the material at temperature T.
Rewrite (4) in expanded form:

$$d\varepsilon_{xx} = d\left(\frac{\sigma_{xx} - \sigma}{2G} + K\sigma\right) + (\sigma_{xx} - \sigma)d\lambda, \ d\varepsilon_{yy} = d\left(\frac{\sigma_{yy} - \sigma}{2G} + K\sigma\right) + (\sigma_{yy} - \sigma)d\lambda,$$

$$d\varepsilon_{zz} = d\left(\frac{\sigma_{zz} - \sigma}{2G} + K\sigma\right) + (\sigma_{zz} - \sigma)d\lambda, \ d\varepsilon_{xy} = d\left(\frac{\sigma_{xy}}{2G}\right) + \sigma_{xy}d\lambda,$$

(7)

In contrast to the traditional plane deformation, when $\Delta\varepsilon_{zz}(x,y)$ = const, for a refined description of the deformation of the specimen, taking into account the possible increase in longitudinal elongation $\Delta\varepsilon_{zz}$, we present in its form (Bogdanov, 2023; Mahnenko, 1976; Collection: Theory of plasticity IL, 1948):

$$\Delta\varepsilon_{zz}(x,y) = \Delta\varepsilon_{zz}^{0} + \Delta\chi_x x + \Delta\chi_y y, \qquad (8)$$

where unknown $\Delta\chi x$ and $\Delta\chi y$ describe the bending of the prismatic body (which simulates the plane strain state in the solid mechanics) in the Ozx and Ozy planes, respectively, and $\Delta\varepsilon^0_{zz}$ – the increments according to the detected deformation bending along the fibres $x = y = 0$.

Solution Algorithm

Let the nonstationary interaction ((Bogdanov, 2023; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2022; Bogdanov, 2012) occur in a time interval $t \in [0, t_*]$ .Then for every moment of time t:

$$\varepsilon_{xx}^{e} = \frac{\sigma_{xx}-\sigma}{2G} + K\sigma,\ \varepsilon_{yy}^{e} = \frac{\sigma_{yy}-\sigma}{2G} + K\sigma,\ \varepsilon_{zz}^{e} = \frac{\sigma_{zz}-\sigma}{2G} + K\sigma,\ \varepsilon_{xy}^{e} = \frac{\sigma_{xy}}{2G},$$

$$\frac{d\varepsilon_{xx}^{p}}{dt} = \left(\sigma_{xx}-\sigma\right)\frac{d\lambda}{dt},\ \frac{d\varepsilon_{xy}^{p}}{dt} = \sigma_{xy}\frac{d\lambda}{dt},\ \frac{d\varepsilon_{yy}^{p}}{dt} = \left(\sigma_{yy}-\sigma\right)\frac{d\lambda}{dt},\ \frac{d\varepsilon_{zz}^{p}}{dt} = \left(\sigma_{zz}-\sigma\right)\frac{d\lambda}{dt}. \qquad (9)$$

For numerical integration over time, Gregory's quadrature formula (Boli &Waner, 1964) of order $m_1 = 3$ with coefficients D_n was used. After discretisation in time with nodes $t_k = k\Delta t \in [0, t_*]$ $(k = 0, K)$ for each value k we write down the corresponding node values of deformation increments:

$$\Delta\varepsilon_{xx,k} = B_1\sigma_{xx,k} + B_2\sigma_{yy,k} - \beta_{xx},\ \ \Delta\varepsilon_{yy,k} = B_2\sigma_{xx,k} + B_1\sigma_{yy,k} - \beta_{yy},$$
$$\Delta\varepsilon_{zz,k} = \alpha_1\sigma_{zz,k} + \alpha_2(\sigma_{xx,k} - \sigma_{yy,k}) - b_{zz},\ \ \Delta\varepsilon_{xy,k} = B_3\sigma_{xy,k} - b_{xy},$$
$$\beta_{xx} = b_{xx} - \alpha_2(b_{zz} + \Delta\varepsilon_{zz})/\alpha_1,\ \beta_{yy} = b_{yy} - \alpha_2(b_{zz} + \Delta\varepsilon_{zz})/\alpha_1,\ \beta_{zz} = -(b_{zz} + \Delta\varepsilon_{zz})/\alpha_1,$$
$$B_1 = \frac{\alpha_1^2 - \alpha_2^2}{\alpha_1},\ B_2 = \frac{\alpha_2(\alpha_1 - \alpha_2)}{\alpha_1},\ B_3 = \frac{1}{2G} + D_0\Delta\lambda_k,$$
$$\alpha_1 = \frac{1}{3}\left(K + \frac{1}{G} + 2D_0\Delta\lambda_k\right),\ \ \alpha_2 = \frac{1}{3}\left(K - \frac{1}{2G} - D_0\Delta\lambda_k\right),$$
$$b_{ij} = \frac{1}{2G}\sigma_{ij,k-1} + \delta_{ij}\left(K - \frac{1}{2G}\right)\sigma_{k-1} - \sum_{n=1}^{m_1} D_n\left(\sigma_{ij,k-n} - \delta_{ij}\sigma_{k-n}\right)\Delta\lambda_{k-n}\ \ (i,j = x,y,z). \qquad (10)$$

The solution of the system (10) gives expressions for the components of the stress tensor at each step [1]:

$$\sigma_{xx,k} = A_1\Delta\varepsilon_{xx,k} + A_2\Delta\varepsilon_{yy,k} + Y_{xx},\ \ \sigma_{yy,k} = A_2\Delta\varepsilon_{xx,k} + A_1\Delta\varepsilon_{yy,k} + Y_{yy},$$
$$\sigma_{zz,k} = -\alpha_2(\sigma_{xx,k} + \sigma_{yy,k})/\alpha_1 - \beta_{zz},\ \ \sigma_{xy,k} = A_3\Delta\varepsilon_{xy,k} + Y_{xy},$$
$$Y_{xx} = A_1\beta_{xx} + A_2\beta_{yy},\ \ Y_{yy} = A_2\beta_{xx} + A_1\beta_{yy},\ \ Y_{xy} = A_3 b_{xy},\ \ A_3 = 1/B_3,$$
$$A_1 = B_1\Big/\left(B_1^2 - B_2^2\right),\ \ A_2 = -B_2\Big/\left(B_1^2 - B_2^2\right). \qquad (11)$$

Function Ψ = 1/(2G) + $\Delta\lambda$, which is characterizing the yield condition, taking into account (8), (9), (11) is:

$$\psi = \begin{cases} \dfrac{1}{2G} & (f < 0) \\ \dfrac{1}{2G} + \dfrac{3\Delta\varepsilon_i^p}{2\sigma_i} & (f = 0, df = 0), \\ (f > 0 - \text{inadmissible}) & \end{cases} \qquad (12)$$

$$\Delta\varepsilon_i^p = \frac{\sqrt{2}}{3}\left(\left(\Delta\varepsilon_{xx}^p - \Delta\varepsilon_{yy}^p\right)^2 + \left(\Delta\varepsilon_{xx}^p - \Delta\varepsilon_{zz}^p\right)^2 + \left(\Delta\varepsilon_{yy}^p - \Delta\varepsilon_{zz}^p\right)^2 + 6\left(\Delta\varepsilon_{xy}^p\right)^2\right)^{1/2},$$
$$\Delta\varepsilon_{xx}^p = \Delta\varepsilon_{xx} - \Delta\varepsilon_{xx}^e,\ \ \Delta\varepsilon_{yy}^p = \Delta\varepsilon_{yy} - \Delta\varepsilon_{yy}^e,\ \ \Delta\varepsilon_{xy}^p = \Delta\varepsilon_{xy} - \Delta\varepsilon_{xy}^e,\ \ \Delta\varepsilon_{zz}^p = \Delta\varepsilon_{zz} - \Delta\varepsilon_{zz}^e,$$
$$\Delta\varepsilon_{xx}^e = \frac{1}{2G}\sigma_{xx} + \left(K - \frac{1}{2G}\right)\sigma,\ \ \Delta\varepsilon_{yy}^e = \frac{1}{2G}\sigma_{yy} + \left(K - \frac{1}{2G}\right)\sigma,$$
$$\Delta\varepsilon_{zz}^e = \frac{1}{2G}\sigma_{zz} + \left(K - \frac{1}{2G}\right)\sigma,\ \ \Delta\varepsilon_{xy}^e = \frac{1}{2G}\sigma_{xy},\ \ \sigma = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3}.$$

Considering when calculating the value $\Delta\varepsilon^p_{zz}$, we found that its impact is so small that without reducing the accuracy of calculations can be considered $\Delta\varepsilon^p_{zz} = 0$.

To take into account (Bogdanov, 2023; Mahnenko, 1976) the physical nonlinearity contained in conditions (12), the method of successive approximations is used, which makes it possible to reduce a nonlinear problem to a sequence of linear problems (Bogdanov, 2023; Mahnenko, 1976):

$$\psi^{(n+1)} = \left\{\psi^{(n)}p + \frac{1-p}{2G},\ \text{if}\ \sigma_{iS} < -Q;\ \psi^{(n)},\ \text{if}\ -Q < \sigma_{iS} < Q;\ \psi^{(n)}\frac{\sigma_i^{(n)}}{\sigma_S(T)},\ \text{if}\ \sigma_{iS} > Q\right\},$$
$$\sigma_{iS} = \sigma_i^{(n)} - \sigma_S(T), \qquad (13)$$

where Q – the value of the largest deviation of the stress intensity $\sigma_i^{(n)}$ in step n from the strengthened yield strength; n – is the approximation number.

Unknown (Boli & Waner, 1964), $\Delta\chi x$, $\Delta\chi y$ and $\Delta\varepsilon^0_{zz}$ in (8) are determined from the conditions of equilibrium of even with respect to x normal stresses σ_{zz}.

$$\iint\limits_{\Sigma} \sigma_{zz}(x,y)\rho\,dxdy = M_\rho,\ (\rho = 1, x, y), \qquad (14)$$

When $M_l = M_x = M_y = 0$; where M_l – projection on the axis Oz of the main vector of contact stresses, and M_x, M_y – corresponding projections of the main moment of the forces acting on the resistance (no torsion, as noted). Given the symmetry of the problem and $\sigma_{zz}(x,y) = \sigma_{zz}(-x,y)$ this equation in case of $p = x$ is satisfied automatically.

If we substitute (8) and (12) in (14), taking into account the symmetry of the integration domain with respect to x and the even of functions $\sigma_{xx,k}$, $\sigma_{yy,k}$, b_{zz} , we have $\Delta\chi x = 0$. A system of linear algebraic equations is obtained for the calculation of $\Delta\varepsilon^0_{zz}$, $\Delta\chi_y$:

$$\Delta\varepsilon_{zz}^0 L_{\rho 1} + \Delta\chi_y L_{\rho y} = \bar{M}_\rho,\ (\rho = 1, y),$$
$$\bar{M}_\rho = \iint\limits_{\Sigma} \frac{\alpha_2\left(\sigma_{xx} + \sigma_{yy}\right) - b_{zz}}{\alpha_1}\rho r\,dxdy,\ \ L_{\rho r} = \iint\limits_{\Sigma} \frac{\rho r\,dxdy}{\alpha_1},\ \ (r,\rho = 1, x, y). \qquad (15)$$

The stresses and strains used above were determined for each unit cell from the numerical solution at each point in time $t_k = k\Delta t$.

Numerical Solution

For this problem the explicit scheme of the finite difference method was used with a variable partitioning step along the axes Ox (M elements) and Oy (N elements). The step between the split points was the smallest in the area of the layers contact and at the boundaries of the computational domain. Since the interaction process is fleeting, this did not affect the accuracy in the first thin layer, areas near the boundaries, and the adequacy of the contact interaction modelling.

The use of finite differences (Hemming, 1972) with variable partition step for wave equations is justified in (Zukina, 2004), and the accuracy of calculations with an error of no more than $O\left((\Delta x)^2+(\Delta y)^2+(\Delta t)^2\right)$ where Δx, Δy and Δt – increments of variables: spatial x and y and time t. A low rate of change in the size of the steps of the partition mesh was ensured. The time step was constant.

The resolving system of linear algebraic equations with a banded symmetric matrix was solved by the Gauss method according to the Cholesky scheme.

In (Weisbrod & Rittel, 2000), during experiments, compact samples were destroyed in 21–23*ms*. The process of destruction of compact specimens from a material of size and with contact loading as in (Weisbrod & Rittel, 2000) was modelled in a dynamic elastoplastic formulation as plane strain state, considering the unloading of the material and the growth of a crack according to the local criterion of brittle fracture. The samples were destroyed (Bogdanov, 2023) in 23*ms*. This confirms the correctness and adequacy of the developed formulation and model.

The thickness of steel layers are the same $h_1 = h_2 = 0.5mm$. Figs. 2 – 10 show the results of calculations of two layers specimens with a hardening factor of the material $\eta^* = 0{,}05$. The first upper and third layers have made from hard steel. The second and fourth main layers have made from quartz glass. Contacts between four layers are an ideal. Calculations were made at the following parameter values: temperature $T = 50^oC$; $L = 60mm$; $B = 10mm$; the contact zone was equal $a = 2A = 0.05mm$, $\Delta t = 3.21\text{-}10^{-8}\,s$; $p_{01} = 8MPa$; $p_{02} = 10MPa$; $M = 62$; $N = 115$. The smallest splitting step was 0,005 mm, and the largest 2, 6 mm ($\Delta x_{\min} = 0.005\ mm$; $\Delta y_{\min} = 0.02\ mm$; (only the first layer); $\Delta x_{\max} = 2.6\ mm$; $\Delta y_{\max} = 0.4\ mm$). The cells of the grid of partitions by coordinate x with the largest size were at the outer side boundaries of the sample, which are located far from the contact area.

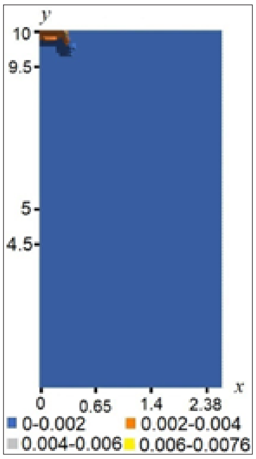

Fig. 2. 4 layers. Odquist parameter K when $t = t_1$

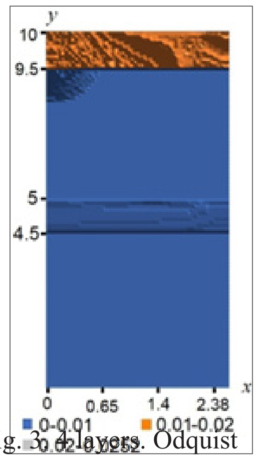

Fig. 3. 4 layers. Odquist parameter K when $t = t_2$

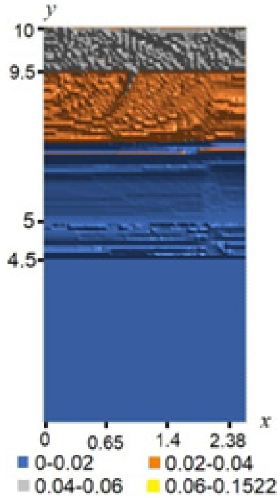

Fig. 4. 4 layers. Odquist parameter K when $t = t_3$

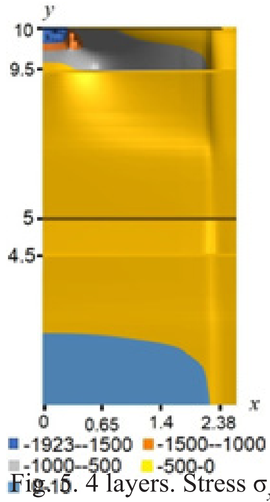

Fig. 5. 4 layers. Stress σ_{xx} when $t = t_1$

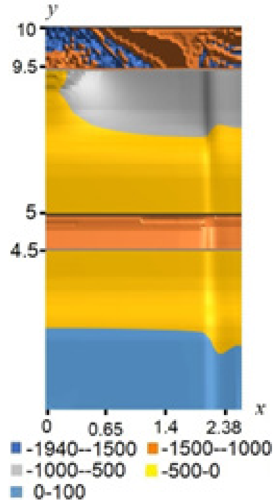

Fig. 6. 4 layers. Stress σ_{xx} when $t = t_2$

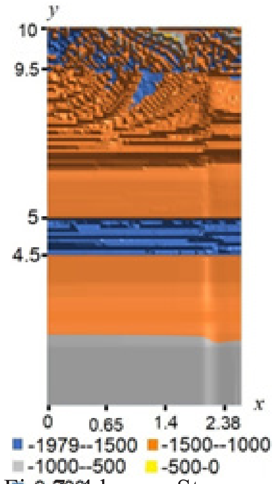

Fig. 7. 4 layers. Stress σ_{xx} when $t = t_3$

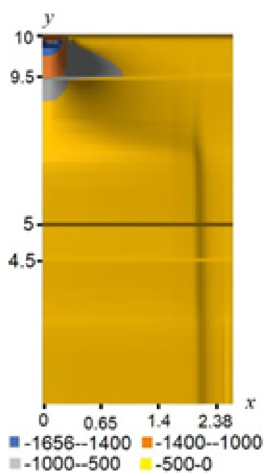

Fig. 8. 4 layers. Stress σ_{yy} when $t = t_1$

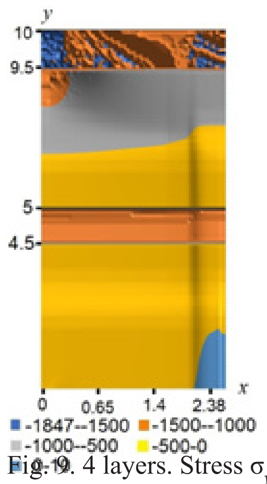

Fig. 9. 4 layers. Stress σ_{yy} when $t = t_2$

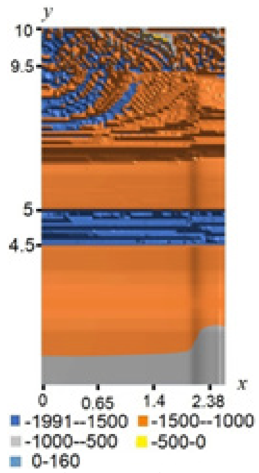

Fig. 10. 4 layers. Stress σ_{yy} when $t = t_3$

Fig. 11. 2 layers. Odquist parameter K when $t = t_1$

Fig. 16. 2 layers. Stress σ_{xx} when $t = t_3$

Fig. 17. 2 layers. Stress σ_{yy} when $t = t_1$

Fig. 12. 2 layers. Odquist parameter K when $t = t_2$

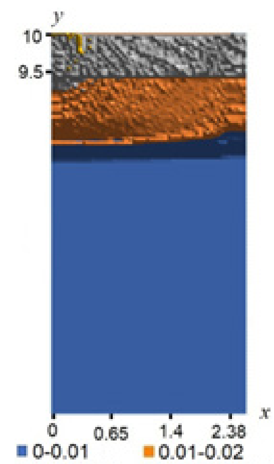

Fig. 13. 2 layers. Odquist parameter K when $t = t_3$

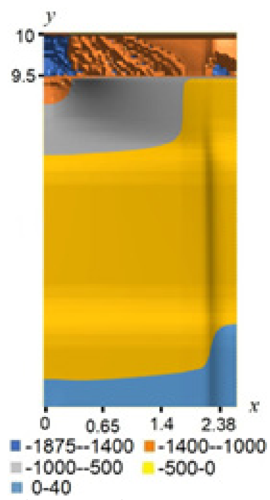

Fig. 18. 2 layers. Stress σ_{yy} when $t = t_2$

Fig. 19. 2 layers. Stress σ_{yy} when $t = t_3$

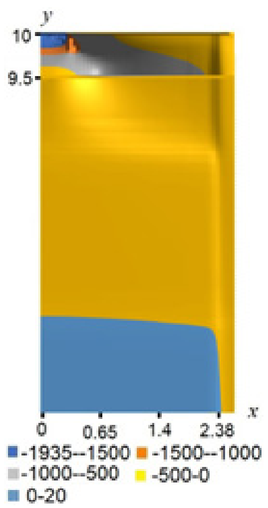

Fig. 14. 2 layers. Stress σ_{xx} when $t = t_1$

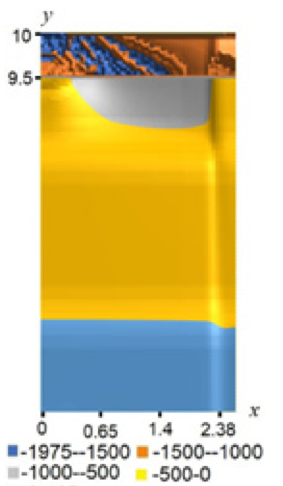

Fig. 15. 2 layers. Stress σ_{xx} when $t = t_2$

Figs. 2 – 4 and 11 – 13, 5 – 7 and 14 – 16, 8 – 10 and 17 – 19 show the fields of the Odquist parameter K, normal stresses σ_{xx} and σ_{yy} at times $t_1 = 2.92\text{-}10^{-6}$ s, $t_2 = 3.95\text{-}10^{-6}$ s and $t_3 = 4.56\text{-}10^{-6}$ s, respectively.

The Odquist parameter as an integral value of the intensity of differential of plastic deformation characterizes the summary plastic deformation.

Figs. 2 – 19 show that the summary plastic deformations in the first upper layer of the steel material in the case of four layers base are bigger than in the steel layer in case of two layers base at the times t_1, t_2 and t_3 , respectively, by 37%, 70% and 78%.

The largest absolute values of normal stresses σ_{xx} and σ_{yy} in the cases of four layers and two layers materials which occur in the first upper layer of the steel material are about the same and have difference by about 3%. These Figs. show areas where the normal stresses in layers are tensile. This is due to the fact that compressive stresses arise in the upper layer quickly and the contact between the layers and the contact of the lower boundary of the lower layer with an absolutely rigid half-space are ideally rigid.

From Fig. 2 – 4, 11 – 13 it can be seen that a thicker layer of the main glass material of the composite base with a thickness of 0.95*mm* in the case of a two-layers base withstands the process of non-stationary interaction and loading better than a thinner upper layer of glass with a thickness of 0.45*mm* in the case of a four-layers composite material.

However, in the case of four layers, the summary plastic deformation, and absolute values of stresses in the upper first steel layer and the first upper glass layer located between two thin steel layers are significantly larger than in investigated two-layers base with twice thicker glass layer. Perhaps the layers of steel located more deep in the composite base can be made thinner compared to the first upper layer of steel. It also seems logical to use thinner layers of glass at a greater depth of the base. This will lighten the composite material. These assumptions require further investigation.

Conclusions

The developed methodology of solving dynamic contact problems in an elastic-plastic dynamic mathematical formulation makes it possible to model the processes of impact, shock, and non-stationary contact interaction with the four-layers elastic composite base adequately. In this work, the process of impact on a four-layers base, consisting of two two-layers bases which consist of an upper thin layer of metal and a lower main layer of glass, is adequately modelled, and investigated. The fields of summary plastic deformations and normal stresses arising in the base are calculated and compared to the corresponding values from the corresponding problem of plane strain state of two-layers composite base. The upper metal layer of the composite four-layers base takes on the main load. The results obtained make it possible to design the narrow strips of new composite reinforced armed materials. Such a four-layer reinforced composite material can be used as a wide range of needs of modern industry.

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