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The Auxiliary Problem for Solving Dynamic Contact Problems Considering Tangential Movement

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Abstract

An auxiliary plane problem on the action of a stamp in the tangential direction is solved. The kernel of the integral relationship between the harmonics of tangential stresses and the harmonics of the tangential component of the velocity vector is obtained. Based on this, a resolving infinite system of Volterra integral equations of the second kind with respect to the unknown harmonics of the tangential component of the velocity vector is derived. The obtained solutions make it possible to solve impact problems when the striker moves at an arbitrary angle and more accurately calibrate the computational process for solving contact problems in a dynamic elastic-plastic formulation.

Keywords: Plane, tangential, impact, elastic, contact, dynamic.

Introduction

This paper provides a solution to an auxiliary problem, as a result of which the kernel of the integral expression for the component of the normal component of the velocity vector through the normal stress component on the surface of the elastic half-space is obtained. This makes it possible to formulate a resolving infinite system of integral Volterra equations of the second kind with respect to unknown harmonics of tangential stresses. This article is an addition to (Bogdanov, 2023; Popov, 1989; Kubenko et al., 1995; Kubenko & Popov, 1988; Bogdanov, 2023; Golovchan, et al; 1986).

The use of a generalized approach to solving dynamic contact problems in an elastic-plastic formulation makes it possible to use it to solve contact problems for a body of arbitrary shape, which is subjected to an arbitrary distributed over the contact zone or shock loading.

In Bogdanov (2023); Bogdanov (2022); Bogdanov (2022); Bogdanov (2023); Bogdanov (2023), a new approach to solving the problems of impact and nonstationary interaction in the elastoplastic mathematical formulation was developed.
In Bogdanov (2023); Bogdanov (2022); Bogdanov (2022); Bogdanov (2023); Bogdanov (2023) dynamic interaction process of plane hard body and two layers reinforced composite material was investigated and the fields of summary plastic deformations and normal stresses arising in the base are calculated using plane strain Bogdanov (2023); Bogdanov (2022); Bogdanov (2022); Bogdanov (2023); Bogdanov (2023) and plane stress Bogdanov (2023); Bogdanov (2022); Bogdanov (2023) states models. In Bogdanov (2022), results depend on the size of the area of an initial contact between the impactor and the upper surface of the base and depend on the thickness of the top metal layer of the composite base. In Bogdanov (2022), results were calculated depending on the material of top layer of the composite base. Composite bases reinforced by steel, titanium and aluminium top layers were investigated. In Bogdanov (2023), the problem of plane strain state of four-layer composite reinforced base was solved.

In contrast from the work (Popov, 1989; Kubenko et al., 1995; Kubenko & Popov, 1988; Bogdanov, 2023; Bogdanov, (2023)), in these papers, we investigate the tangential impact process.

Problem Formulation

It was solved problem when absolute hard cylinder moving to the surface of an elastic half-space $z \geq 0$ collides with an elastic half-space at a time $t = 0$ (Fig. 1). The initial contact was along cylinder generatrix. The cylinder begins to penetrate into the elastic medium at a rate $v_{1T}(t)$, and the initial rate of penetration $V_{10} = v_{1T}(0)$. It was associated with the shell, a moving cylindrical coordinate system: $r\theta y' : \theta$ is –the polar angle, which is postponed from the positive direction of the axis Oz,. The axis $O'y'$ coincides with the axis of the cylinder. A fixed Cartesian coordinate system xyz was connected with a half-space, so that the Oz axis is directed inwards, the Ox axis – is on the surface of the half-space and the Oy axis – is parallel to the cylinder generatrix. It was assumed that there are no tangential movements during the penetration process.

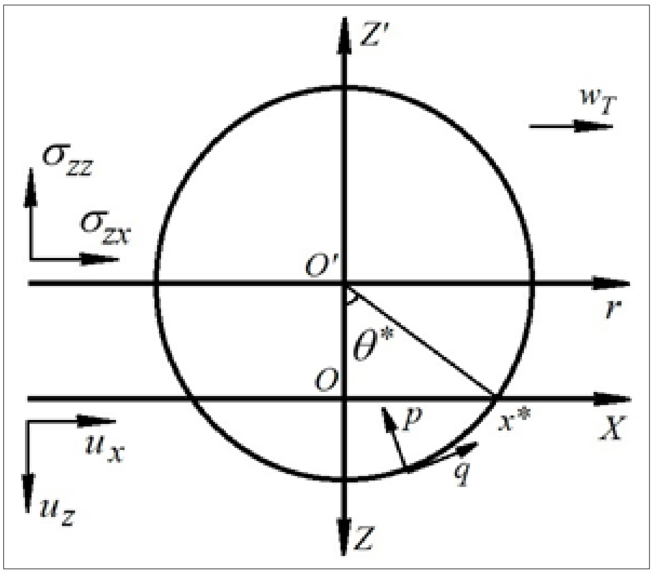

Figure 1: Calculation scheme

In the problem under consideration, an idealized tangential movement of the striker is assumed, which is symmetrical relative to the plane $x = 0$, which causes a stress-strain state symmetrical relative to this plane. The purpose of this work is to derive the kernel of the integral relationship between the harmonics of the tangential stress and the harmonics of the tangential component of the velocity vector and test it by calculating the solution to this problem.

The physical properties of the half-space material are characterized by elastic constants: the modulus of volume expansion K, the shear modulus μ and the density ρ. The elastic medium with constants K, μ, and ρ will correspond to the hypothetical acoustic medium with the same constants K, μ, and ρ; thus $\mu = 0$. By C_p, C_S and C_0 we mean the speed of longitudinal and transverse waves in an elastic half-space, as well as the speed of sound in a hypothetical acoustic medium corresponding to the considered half-space.

Let into the notation:

$$\beta^2 = \frac{C_S^2}{C_0^2} = \frac{\mu}{K}, \ \alpha^2 = \frac{C_p^2}{C_0^2} = \left(1 + \frac{4\mu}{3K}\right),$$

$$C_0^2 = \frac{K}{\rho}, \ b^2 = \frac{\beta^2}{\alpha^2} = \frac{3\mu}{3K + 4\mu}. \quad (1)$$

We introduce dimensionless variables:

$$t' = \frac{C_0 t}{R}, \ x' = \frac{x}{R}, \ z' = \frac{z}{R}, \ u_i' = \frac{u_i}{R},$$

$$\sigma_{ij}' = \frac{\sigma_{ij}}{K}, \ v_T' = \frac{v_T}{C_0}, \ w_T' = \frac{w_T}{R}, \quad (2)$$

$$p' = \frac{p}{KR}, \ q' = \frac{q}{KR}, \ M' = \frac{M}{\rho R^2}, \ (i, j = x, z),$$

where $\mathbf{u} = (u_x, u_z)$ is the displacement vector of points of the environment, σ_{zz}, σ_{xz} – are the non-zero components of the stress tensor of the medium, M - is the linear weight of the striker and $v_T(t)$, $w_T(t)$ are the velocity and displacement of the striker as a solid. In the future we will use only dimensionless quantities, so we omit the dash.

The motion of an elastic medium is described by scalar potentials of equations φ and the non -zero component of vector potential ψ satisfying the wave equations:

$$\Delta\varphi = \frac{\partial^2\varphi}{\alpha^2 \partial t^2}, \ \Delta\psi = \frac{\partial^2\psi}{\beta^2 \partial t^2}, \ \Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}. \quad (3)$$

Physical quantities are expressed in terms of wave potentials as follows:

$$u_x = \frac{\partial\varphi}{\partial x} - \frac{\partial\psi}{\partial z}, \ u_z = \frac{\partial\varphi}{\partial z} + \frac{\partial\psi}{\partial x}, \ u_y = 0,$$

$$\sigma_{zz} = (1 - 2b^2)\frac{\partial^2\varphi}{\partial t^2} + 2\beta^2\left(\frac{\partial^2\varphi}{\partial z^2} + \frac{\partial^2\psi}{\partial x \partial z}\right),$$

$$\sigma_{xz} = 2\beta^2 \frac{\partial^2\varphi}{\partial x \partial z} + \frac{\partial^2\psi}{\partial t^2} - 2\beta^2 \frac{\partial^2\psi}{\partial x^2}, \ \sigma_{xy} = \sigma_{yz} = 0, \quad (4)$$

$$\Theta = \sigma_{zz} + \sigma_{xx} = 2(1 - b^2)\frac{\partial^2\varphi}{\partial t^2}, \ \sigma_{xx} = \Theta - \sigma_{zz}.$$

In the coordinate system zOx movement u_z, u_x and stresses σ_{zz} and σ_{zx} at the surface points of the contact area will be written in the form:

$$u_x(t, x, 0) = -w_T(t), \quad |x| \le x^*, \quad (5)$$

$$\sigma_{zx}(t, x, 0) = 0, \quad |x| > x^*, \quad (6)$$

$$\sigma_{zz}(t, x, 0) = 0, \quad |x| < \infty. \quad (7)$$

$w_T(t)$ is the movement of the cylinder as a solid according to low:

$$M\frac{d^2 w_T(t)}{dt^2} = Q(t), \quad (8)$$

$$v_T(t)\big|_{t=0} = V_0, \ w_T(t)\big|_{t=0} = 0, \quad (9)$$

$$Q(t) = 2\int_0^{x^*(t)} \sigma_{zx}(t, x, 0)dx. \quad (10)$$

The initial conditions for the potentials φ and ψ are zero:

$$\varphi\big|_{t=0} = \frac{\partial\varphi}{\partial t}\bigg|_{t=0} = 0, \ \ \psi\big|_{t=0} = \frac{\partial\psi}{\partial t}\bigg|_{t=0} = 0. \quad (11)$$

We assume that the contact region is simply connected, and this statement is equivalent to the fact that normal to the contact area stresses are compressive.

$$\sigma_{zz}\big|_{z=0} < 0, \ \ |x| < x^*(t). \quad (12)$$

Mathematically, we have a non-stationary mixed boundary problem of the theory of elasticity, when displacements are given in the contact region and the rest of the half-space boundary is free of stresses.

Since the impact process is short-lived, the perturbation region at each time t is finite. Limiting the finite time interval of the interaction $(0 \le t \le T)$, we can select a half-space region that by the time T covers the entire perturbation zone. From this point of view, for the times $0 \le t \le T$ elastic half-space can be replaced by elastic half-strip $(|x| \le l;\ z \ge 0)$; perturbations do not reach the limits of time T.

$$l = \alpha T + x^*(T). \tag{13}$$

Thus, for the time being $0 \le t \le T$, and the problem under consideration is reduced to a non-stationary problem for a half-strip under mixed boundary conditions at its end. To represent the displacement vector in the form:

$$u = \operatorname{grad}\varphi + \operatorname{rot}\psi, \quad \operatorname{div}\psi = 0, \tag{14}$$

on the side surface of the semi-strip, we choose the conditions of sliding:

$$u_x\big|_{|x|=l} = 0, \quad \sigma_{zx}\big|_{|x|=l} = 0, \tag{15}$$

We apply to the system of equations (3) the Laplace transform on the variable t (s is the transformation parameter) and the Fourier method of separation of variables, taking into account the even of x potential φ and odd potential φ. Then, in the space of the Laplace transform we obtain the following expressions for wave potentials [6]:

$$\phi\ \ (s,x,z) = \sum^{\infty} A_n(s)\exp(-z(s^2/\alpha^2 + \lambda_n^2)^{1/2})\cos\lambda_n x,$$

$$\psi\ \ (s,x,z) = \sum^{\infty} B_n(s)\exp(-z(s^2/\beta^2 + \lambda_n^2)^{1/2})\sin\lambda_n x. \tag{16}$$

where λ_n are the eigenvalues of the problem, in this case

$$\lambda_n = n\pi / l, \tag{17}$$

In (16), $A_n(s)$ and $B_n(s)$ are determined from the conditions at the boundary. From expressions (16) and relations (4), it follows that the required functions on the surface of the half-space are represented in the form of series according to the system of eigen functions of the problem.

$$\begin{aligned} u_z(t,x,0) &= \sum_{n=0}^{\infty} u_{zn}(t)\cos\lambda_n x, \\ u_x(t,x,0) &= \sum_{n=1}^{\infty} u_{xn}(t)\sin\lambda_n x, \\ \sigma_{zz}(t,x,0) &= \sum_{n=0}^{\infty} \sigma_{zn}(t)\cos\lambda_n x, \\ \sigma_{zx}(t,x,0) &= \sum_{n=1}^{\infty} \sigma_{zxn}(t)\sin\lambda_n x. \end{aligned} \tag{18}$$

Auxiliary Problem

Based on (5), the boundary conditions in the absence of friction in the contact zone can be formulated as follows:

$$\begin{aligned} &\left.\frac{\partial u_z}{\partial t}\right|_{z=0} \equiv V_x = v_T(t), \quad |x| < x^*(t), \\ &\sigma_{zx}\big| = 0, \quad |x| > x^*(t). \end{aligned} \tag{19}$$

Applying to the last equations of the Laplace transform on t and considering (18), we obtain the conditions for the transform at harmonics of the development of the corresponding functions on the surface of the half-space into trigonometric series:

$$su_{xn}^L(s) = V_n^L(s), \quad \sigma_{zzn}^L(s) = 0, \quad (n = \overline{0,\infty}). \tag{20}$$

From (16), (20), using (4), we obtain:

$$\begin{aligned} A_n(s) &= \frac{2\beta^2\lambda_n}{s^3}V_{xn}^L(s), \\ B_n(s) &= -\frac{(s^2 + 2\beta^2\lambda_n^2)V_{xn}^L(s)}{s^3\sqrt{s^2/\beta^2 + \lambda_n^2}}. \end{aligned} \tag{21}$$

We consider (16), (21) in (4) and obtain such a connection between the transformants of the harmonics of the functions σ_{zx} and V_x $(n = \overline{0,\infty})$:

$$\begin{aligned} \sigma_{zxn}^L(s) = &\left(\frac{4\beta^4\lambda_n^2\sqrt{s^2/\alpha^2 + \lambda_n^2}}{s^3} + \right. \\ &\left. + \frac{(s^2 + 2\beta^2\lambda_n^2)^2}{s^3\sqrt{s^2/\beta^2 + \lambda_n^2}} \right) V_{xn}^L(s). \end{aligned} \tag{22}$$

Applying to (22) the inverse Laplace transform and using the convolution theorem, we find the relationship between the harmonics of the tangential component of the velocity vector and tangential stresses on the surface of the half-space:

$$\sigma_{zxn}(t) = \frac{1}{\lambda_n}V_{xn}(t) + \int_0^t V_{xn}(\tau)F(t-\tau)d\tau, \tag{23}$$

where

$$\begin{aligned} F_n(t) = &-\beta J_1(\beta\lambda_n t) + 2\beta^3\lambda_n^2\Big[\Big(2 + \beta^2\lambda_n^2 t^2\Big)\int_0^t J_0(\beta\lambda_n\tau)d\tau - \\ &-2\beta\lambda_n t^2 J_1(\beta\lambda_n t) + \beta^2\lambda_n^2 t^3\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+3)(k!)^2}\left(\frac{\beta\lambda_n t}{2}\right)^{2k}\Big] + \\ &+\frac{2\beta^4\lambda_n^2}{\alpha}\Big[\Big(2 + \alpha^2\lambda_n^2 t^2\Big)\int_0^t J_0(\alpha\lambda_n\tau)d\tau - 2\alpha\lambda_n t^2 J_1(\alpha\lambda_n t) + \\ &+\alpha^2\lambda_n^2 t^3\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+3)(k!)^2}\left(\frac{\alpha\lambda_n t}{2}\right)^{2k}\Big]. \end{aligned}$$

where $J_0(t)$, $J_1(t)$ are the Bessel functions of the first kind of zero and first shape, respectively.

Further, mixed boundary conditions (5) – (7) satisfice. From (5), (23), using $H(x)$, which is – a Heaviside step function, we obtain the following expression for the vertical component of the velocity on the surface of the half-space:

$$\sum_{n=0}^{\infty} V_{xn}(t)\sin\lambda_n x = -\mathrm{H}(x^* - |x|)v_T(t),$$

$$\sum_{n=0}^{\infty} \frac{1}{\lambda_n} V_{xn}(t)\sin\lambda_n x = \qquad (24)$$

$$-\mathrm{H}(|x| - x^*)\sum_{n=0}^{\infty} \sin\lambda_n x \int_0^t V_{xn}(\tau)F_n(t-\tau)d\tau.$$

Representing both parts (24) in the form of series on $\sin\lambda_n x$, we obtain an infinite system of integral equations of Volterra of the second kind relatively unknown tangent velocity harmonics on the surface of the half-space $(n=\overline{0,\infty})$:

$$V_{xn}(t) + \sum_{m=0}^{\infty} \alpha_{mn}(x^*(t)) \int_0^t V_{xm}(\tau)F_m(t-\tau)d\tau = C_n(x^*(t))v_T(t), \qquad (25)$$

$$\alpha_{mn}(x^*(t)) = \frac{1}{2N_{2n}} \int_{x^*}^{l} \cos\lambda_m x \cos\lambda_n x dx,$$

$$C_n(x^*(x)) = -\frac{1}{2N_{1n}} \int_0^{x^*} \sin\lambda_n x dx,$$

$$N_{1n} = \int_0^l \sin^2\lambda_n x dx, \quad N_{2n} = \frac{1}{\lambda_n} N_{1n}.$$

The kinematic condition that determines the half-size of the contact area $x^*(t)$ will be written as follows:

$$w_T(t) - u_z(t,x,0) \begin{cases} = 0, & \text{if} \quad |x| \le x^*(t) \\ < 0, & \text{if} \quad |x| > x^*(t) \end{cases} \qquad (26)$$

Numerical solution will use the same methodology of solving the infinite system of integral equations of Volterra of the second kind (25) considering (26) and (8) – (10) as in [2 – 5, 11].

Conclusions

It was developed the kernel of the auxiliary problem of tangential impact. Using this it was obtained the infinite system of integral equations of Volterra of the second kind. Such problems are important for solving impact problems with not zero tangential component of velocity vector of striker's movement. Using developed formulation, it is possible to simulate more adequately impact, shock processes.

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