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Tessellation of Unlimited Equilateral Rhombuses

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Abstract
A variety of equilateral rhombuses can be tessellated into an unlimited (non-periodic) two-dimensional plane. The ability to tessellate and extend an unlimited number of equilateral rhombuses using an unlimited number of combinations is illustrated here in the context of a standard drawing process.

Introduction

Rhombus-like close-packed combination patterns are often observed in nature, for example, the stripes of snail shells and the spiral core-like distribution of sun flower seeds. Gaining a comprehensive understanding of the rhombus tessellation pattern can aid in understanding the taxonomic intricacies of the similarities and differences of biological growth.

If the acute angle of a rhombus is 90/N degrees, and N is an integer, then there can be N equilateral rhombus whose acute angles are integer multiples of 90/N respectively. A standard drawing method with numerous variables, in which N equilateral rhombus can be tessellated into an infinite number of groups (kinds) of combinations of infinite pattern. This type of drawing rule has three general guidelines.

The first method is the parallel arrangement of diamonds. Different diamonds are arranged in parallel in the same direction then repeat themselves in parallel fashion to form an infinite number of (infinitely large) cluster patterns.

In the second method, some can be rhombus joined into simple polygons first, and subsequently combined into large polygons (the obtuse angle is an integer multiple of 90/N), from which infinite patterns can be created. The third method involved arranging acute angle sizes in a sequential manner to create a basic crescent shape that was densely tessellated with 4N corners. The crescent shape can be repacked into different sickle shapes, and then combined into an infinite spiral pattern.

Similar diamond-shaped patterns are often observed in living organisms, e.g., the stripes of snail shells, the spiral core of sunflower seeds, and the needles of cacti. A systematic understanding of all the possibilities of rhombic pavement patterns can help us to understand the categorisation of organisms in terms of their growth differences and similarities.

A book that shows how to create an infinitely large (no periodicity) two-dimensional flat pattern using various equilateral rhombus tessellations has been published by the author (Penrose, R. (1974). Two dimensional tessellation art and illusion pattern by Kung Chung-yuen, Hung Lam Book Co. (Honglin Book Co., Ltd.)). This type of standard drawing rules with very few elements, minimal variables, infinite combinations, and unbounded derivation (expansion) is worthy of further detailed explanation. Text descriptions are vague, boring and easier to understand than pictures. If the acute angle of a rhombus is 90/N degrees, and N is an integer, then there can be N equilateral rhombuses whose acute angles are integer multiples of 90/N respectively. A standard drawing method with numerous variables, in which N equilateral rhombus can be tessellated into an infinite number of groups (kinds) of combinations of infinite pattern. There are four general rules of drawing, which are described hereafter.

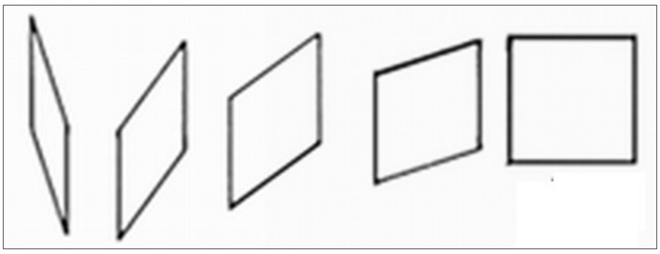

Figure 1: As N is 5 used for illustration. The five acute angles of N for 5 are 18, 36.54, 72, and 90 degrees.

First Rule: Arbitrary Parallel Combinations

This is simplest and easiest to understand such that five equilateral rhombuses are arbitrarily arranged in parallel in the same direction (as shown in Figure 2). This configuration can be semi-symmetrical left and right, but cannot be symmetrical in four directions. Readers can further derive any rhombus to infinity according to this tessellation rule.

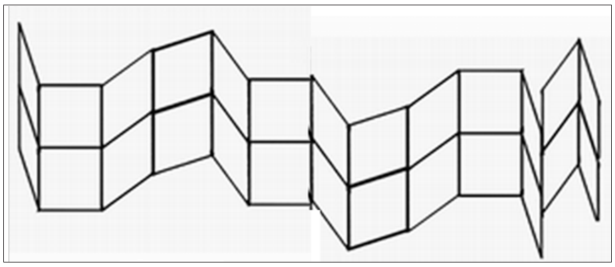

Figure 2: Equilateral rhombuses are arbitrarily arranged in parallel in the x-direction and alsoin y-direction.

The Second Rule: Radial Configuration

Some rhombuses can be densely packed to form a simple 90-angle configuration, then combined into a 360-degree configuration, and then the first parallel arrangement rule can be used to derive an infinite radial pattern (Figure 2a).

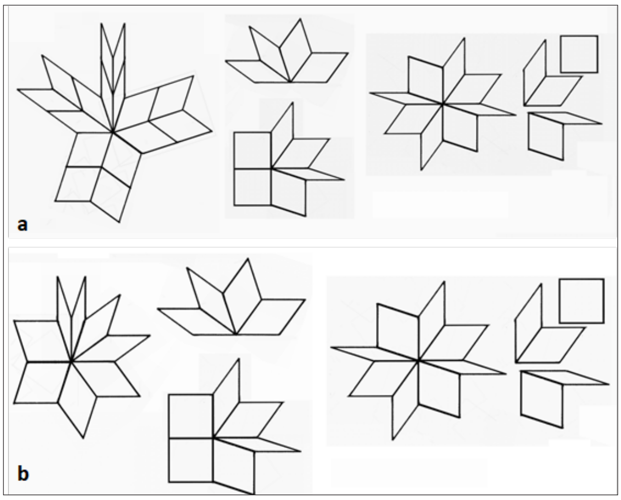

Figure 3: (a) array of five rhombus in different way and (b) can be extended to infinityradiuly.

The Rhird Rule: Stacking of Building Blocks

The third rule is the more traditional block stacking from small to large blocks. Figure 3 is an illustration of a basic polygon. Basic polygons can be stacked to form a partially overlapping polygon, which can then be combined to form larger and infinitely larger polygons. As in Figure 4, all corners must be integer multiples of 18 degrees (the smallest acute angle). It can be stacked into a four-fold symmetry, but it is more difficult.

The Fourth Rule: Crescent Moon Swing Closure

The fourth rule is the most creative; all rhombuses are arranged in parallel and sequential order according to the size of their acute angles, as shown in Figure 5 (which is an illustration of a crescent). This crescent shape has two acute angles of 18 degrees, 18 obtuse angles of 162 degrees, and 20 sides of the same length. The minimum acute angle and the sum of all obtuse angles of the crescent side must be 180 degrees. N can be 1, 2, 3, 4, 5... to infinity, and 90/N can be infinit decimals (When N is equal to infinity, the acute angle of the crescent is zero, and the other obtuse angles are 180 degrees. At this time, the two curves of the crescent are closer to forming a semicircle.

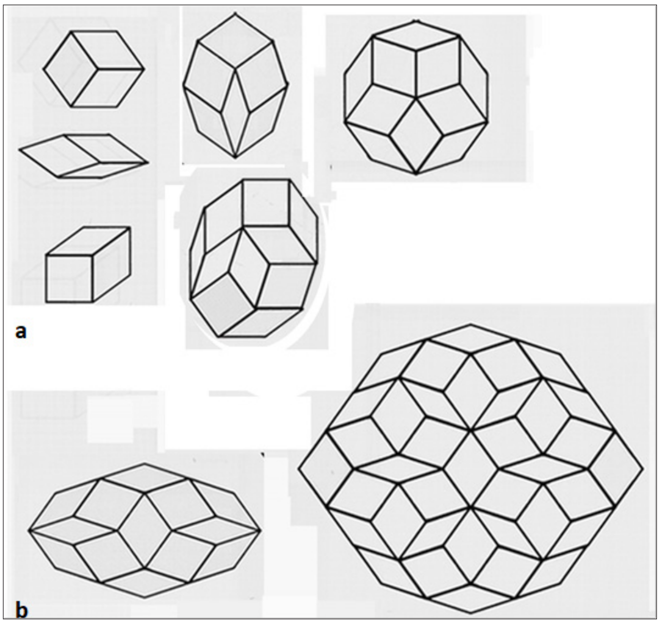

Figure 4: (a) Basic polygons and (b) various larger equilateral polygons by try error.

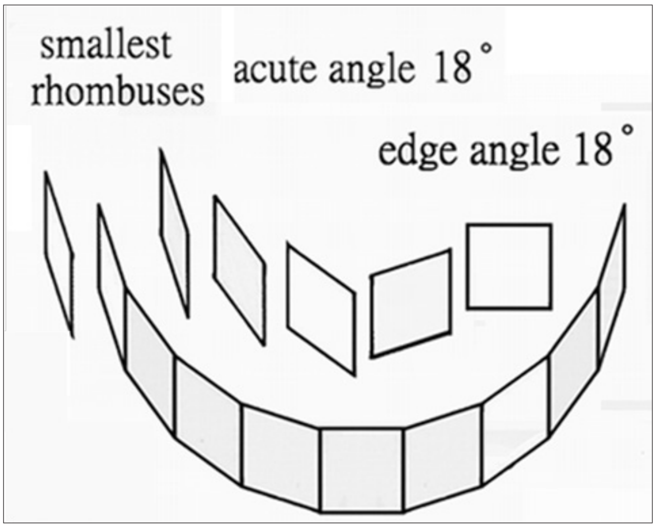

Figure 5: This crescent shape has 2 acute angles of 18 degrees and 18 side angles of 162 degrees, with 20 equal angle sides.

Sickle-shaped Configuration

Two or more basic crescent shapes can be rotated relative to each other by an integer multiple of the minimum acute angle, and then closely (attached) to each other to form various sickle-shaped configurations, as shown in Figure 6.

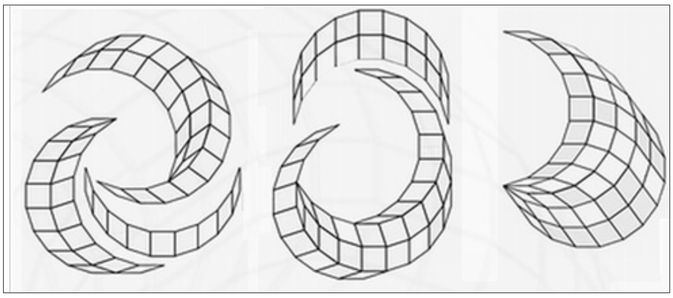

Figure 6: An example of a sickle-shaped configuration.

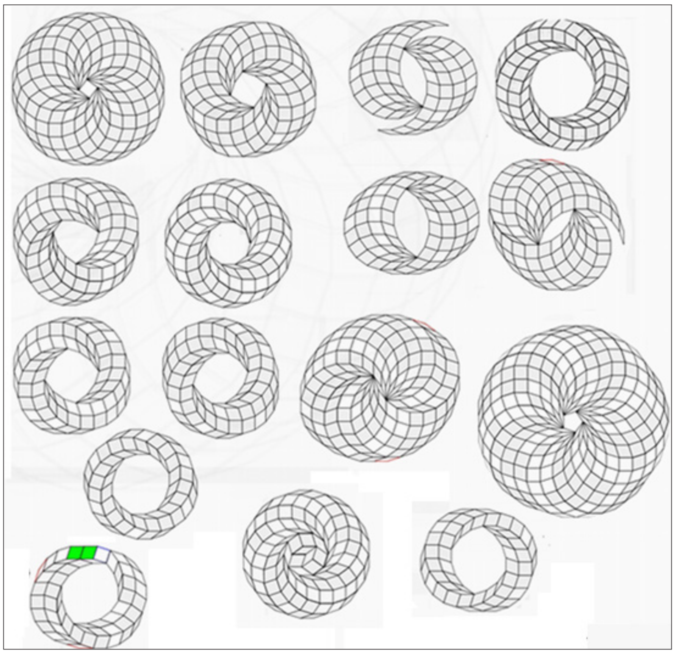

Figure 7: Circular, oval or ring configuration.

The sickle-shaped configurations in the figure above can be expanded to form circular, oval, or ring configurations, as shown in Figure 7. The hollow of the ring can sometimes be perfectly filled with a proper polygon.

It does not have to be five rhombuses to form a crescent. Two, three or four equilateral rhombuses can also form a crescent with different acute angles. These crescent shapes can be configured in spiral turns or different sickle shapes, as shown in Figure 8. These small configuration variations contribute to the diversity of the final pattern, making it easier for the reader to understand the so-called infinite combination connotation.

These configurations can be further combined into specific types of two-dimensional tessellation patterns. There are many ways to derive the configuration to infinity. The most systematic approach involves selecting any polygon as the core and then using crescent-shaped adjacent dense paving to derive it to infinity, as shown in Figure 9.

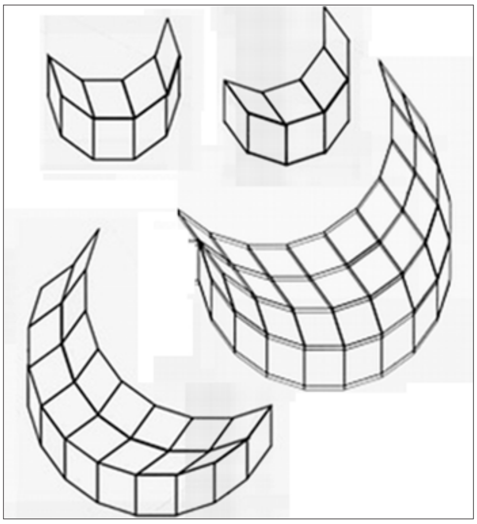

Figure 8: Crescent shapes can be configured in spiral turns or different sickle shapes.

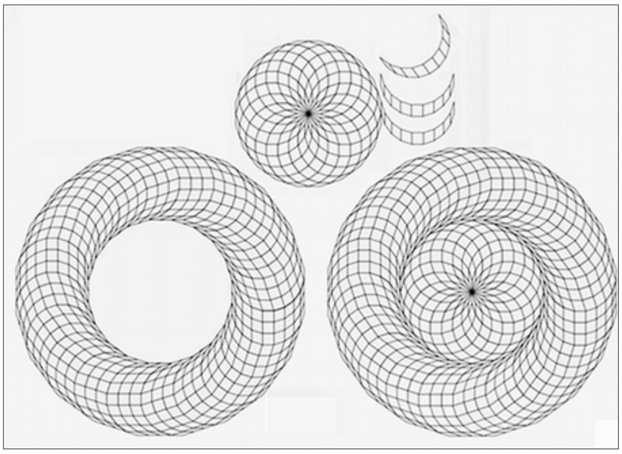

Figure 9: Crescent-shaped adjacent dense paving.

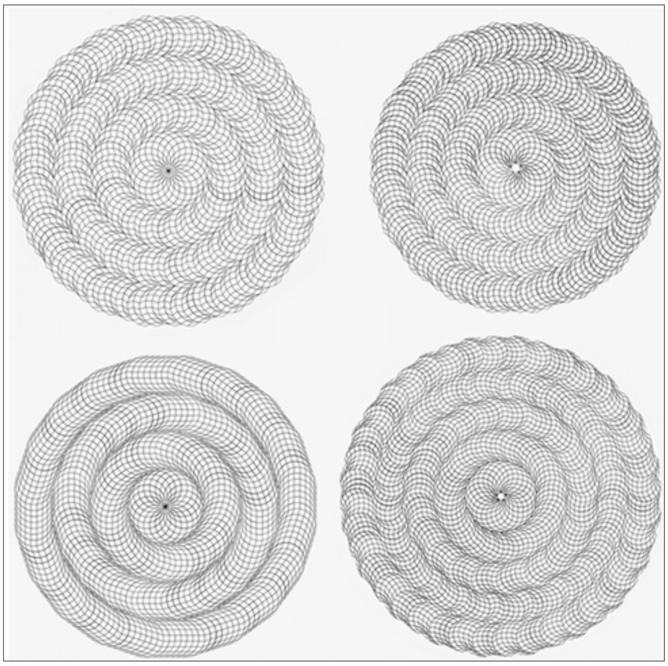

Figure 10: A specific type of two-dimensional tessellation pattern with one circular ring coexisted of all five rhombus.

With the spiral core of sun flower seed as the core, the first ring is produced, as shown in Figure 10. Using the same steps, the second ring, the third ring, and an infinite number of concentric rings can be derived.

The spiral core of sun flower seed is four-dimensional symmetrical. It can be encircled by an elongated crescent shape to form an infinite pattern, which can be made into an infinite four-dimensional symmetrical pattern. As shown in Figure 11, different rhombuses can be added in the appropriate wrapping process to form an elongated crescent-shaped fugure.

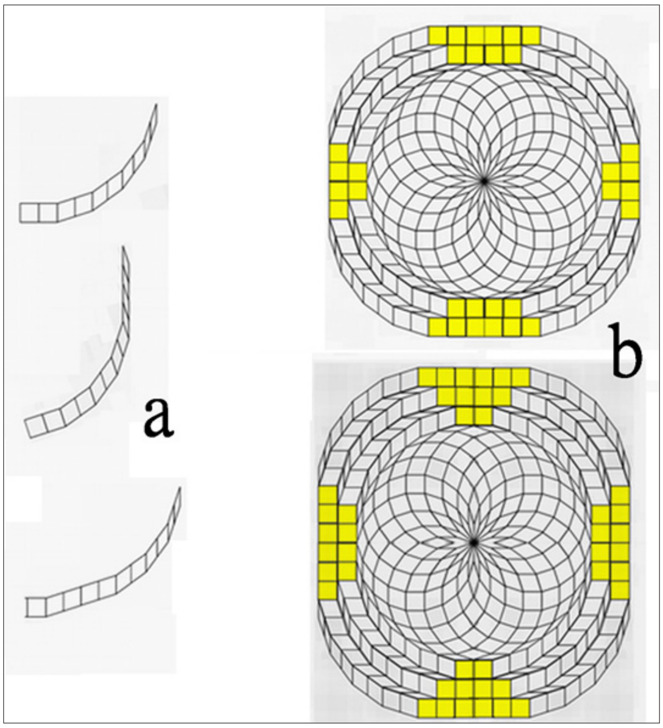

Figure 11: (a) Properly lengthen the sickle shapes, and (b) demonstration of a four way symmetry tiling.with all five different rhmbous.

Finally, I will use twodifferent rhombuses, with acute angle 36 degrees and 72 degrees, to make a swirl type aperiodic tiles to infinite as a comparison with Penrose tiling (ref. 1). As shown in Fig 12 (a, b), the left isde one is five-fold rotaional systemtric , but not mirror symmetry, and left side one, is no symmetric at any orientation.

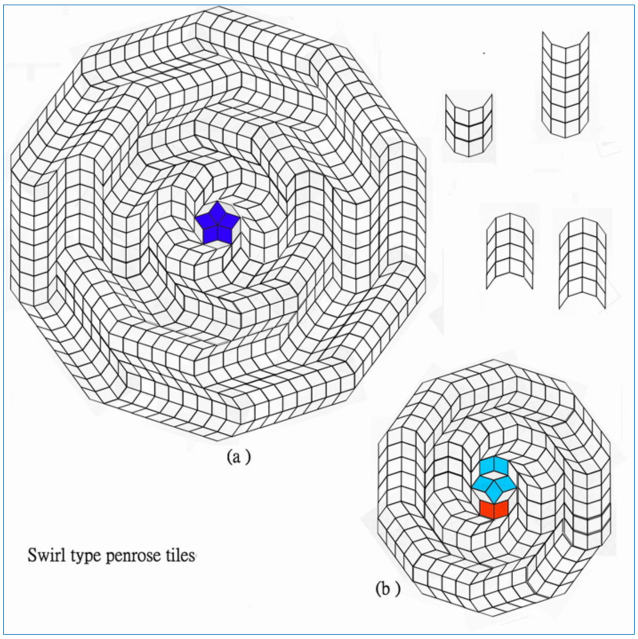

Figure 12: Penrose tiling of swirl type, (a) five-fold rotational symmetric (b) non symmetric.

References

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