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The Formation of Symmetric and Asymmetric Binary-Decagon Penrose Tiles By Conjugate Coupling Process

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Abstract

In this article, we made some fivefold symmetric and asymmetric Penrose tiles in the shape of pentagons, which consist of only two types of a- and b-type decagons. These tiles can be conjugate coupled together to create symmetrical five-fold tiles as well as asymmetry Penrose tiles. They are all defect-free, and still remained consisted of only, type-a and type-b (as still)a binary tiles system.

Keywords: Penrose tiles, conjugate coupling, binary tile.

Introduction

Previously, we have demonstrated that Penrose tiles can be translational expended with type-a and type-b two decagons (Penrose 1974), and even translated periodically expanded with together of all six different decagons (Kung, 2024). It has been also demonstrated that using a five-fold symmetry tile of pentagonal Penrose tile, consisted of only type-a and type-b decagons, can be unlimited coupled and still consisted only two types of decagons as described above. But when these same tiles can always generated defect or other types of decagons during the coupling process for constructing a asymmetric tile. (Kung, 2024). In this article, we have made some pentagon-shaped fivefold symmetric and asymmetric Penrose tiles, which consist of only two types, type-a and type-b decagons. These tiles can be combined to create five-fold symmetrical tiles as well as asymmetrical Penrose tiles. They are all defect free and still consist of only type a and type b (still) binary tile systems.

Result and Discussion

Two different types of pentagonal trinary Penrose tiles, consisted of three types of decagons, can be completed conjugate coupling in different depth (Kung,2024), This pair of this tiles in reference (Kung,2024) can be converted into a pair of binary Penrose tiles, as show in fig 1(a, b). The graph in Figure 1(a) has 5 decagons on each side and the star shape in the type-a decagon are marked as red. The star on the five vertices on each tile, are all pointing up-ward. And this Penrose tile is slightly outward relative to the regular pentagon shape, defined as the inflated (swallow) type. The center of this Penrose tile is a single type-a decagon, marked in dark blue, and defined the tile as a positive Penrose tile.

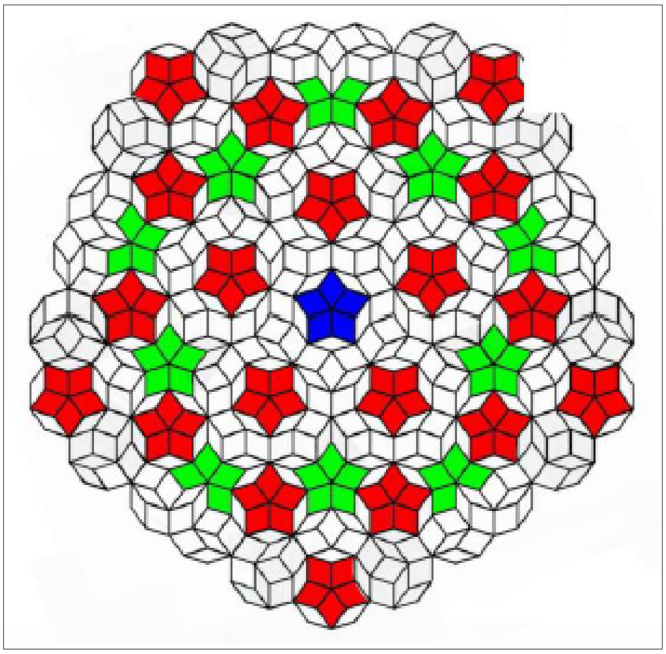


Figure 1 A : Symmetry Penrose tile of Pentagon shape consisted of type-a and type-b decagons (a) swollen type

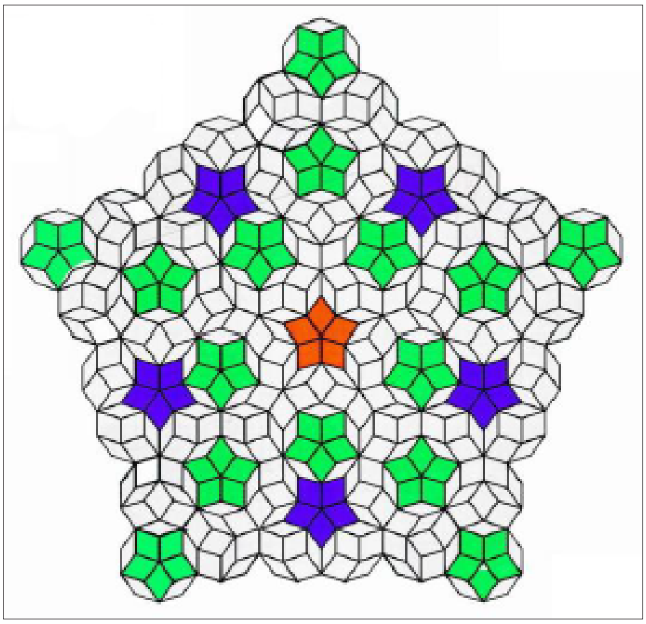


Figure 1 B: slim type.

In the fig.1 b, we used different colors to mark the decagons in order to avoid the confusion during the conjugate coupling. The green color marked on the star of all type-a decagon. All the star on the five vertices are pointing downward, and the arrangement of the five decagons on each side is slightly inward than the regular pentagonal shape, defined as a slim type Penrose tile. The middle star made up of (parts of) five type-b decagons is marked in orange color, defined as negative Penrose tiles. Both positive and negative Penrose tiles are five-fold symmetry (rotation 72 degree around the center of the dark blue star or orange star return to origin) around the center of the colorful star. The decagonal arrays on each side of the positive and negative pentagonal Penrose are arranged in the same way and therefore match perfectly without mismatched in the seam (region) area (Kung, 2024). Based on the pair of Penrose tile described, a set of asymmetric Pentagonal type Penrose tile were made, positive type is shown in fig 2 a b c d, and negative pentagonal type Binary Penrose tiles with the same outer decagon just the only the same one as shown in fig 1b. (In other words. there is no asymmetry binary Penrose tile of the size that matched the original Penrose tile with five-decagons one side).

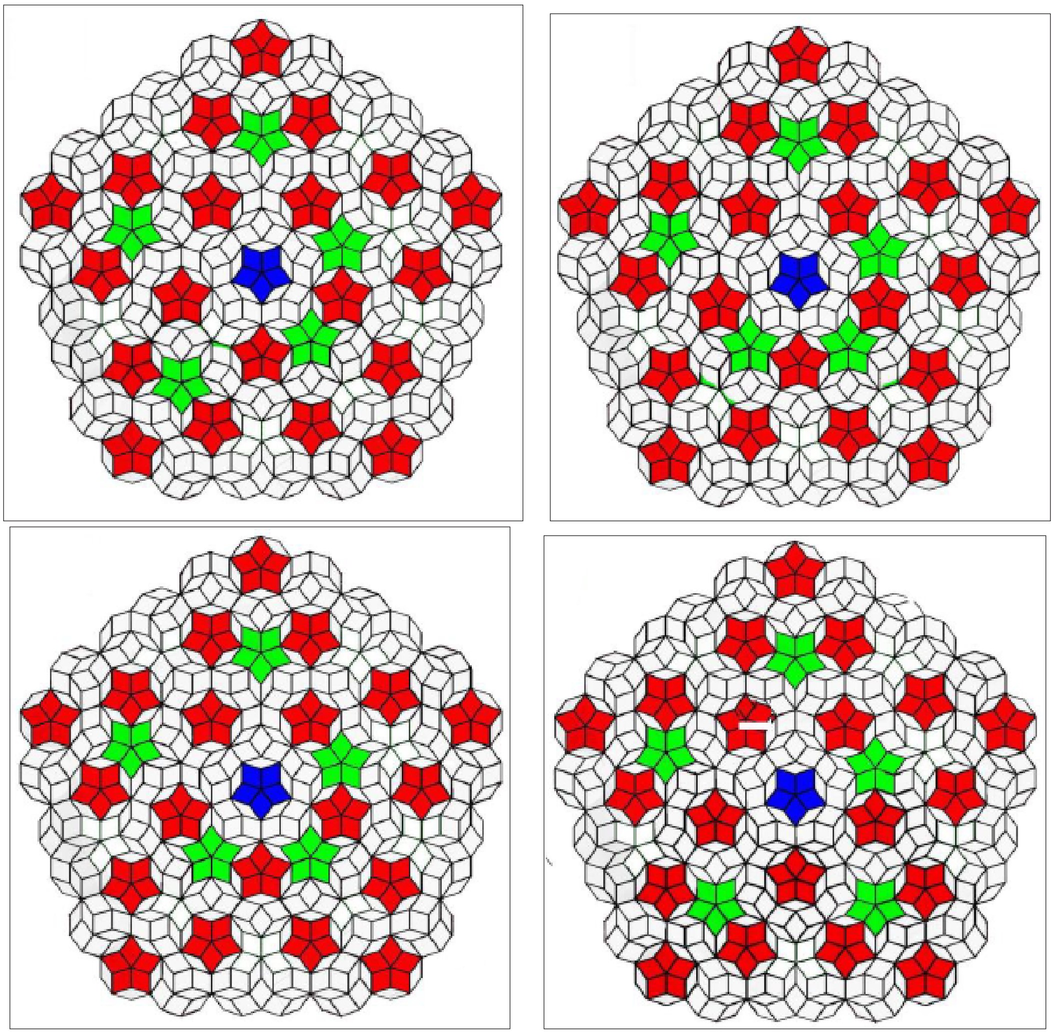


Figure 2A : set of asymmetry swollen type binary tiles with five decagons each side.

This article demonstrates the process of expanding (expanding) Penrose tiles by conjugate two types of Penrose tiles (slim and bloated). In particular, these tiles consist only of type–a and type-b decagons, which makes it more difficult to construct infinite Penrose tiles compare with the tiles consisted of more than two decagons. **(Mathematically speaking, fewer variables would produce a finite change to meet the requirement of a binary tile)**. It is necessary to show how to do this in an infinitely coupled process while maintaining the tile as a defect-free binary Penrose.

The Detail Procedures are Showing as Follows

First, taking the positive Penrose diagram (five decagons on each side) in Figure 1a as the center, using the graph in Figure 1 b, apply skin (outer-ring)coupling (Kung, 2024) (bracketing) five times surrounding to the graph in Figure 1a, and the result is shown in Figure 3a., in the same way, take the graph in Figure 1b as the center, the graph in the Figure 1a surround coupling five times on the graph in the Figure 1b, results a graph as shown in Figure 3b, Both graphs in fig 3(a, b) are existed with empty area.

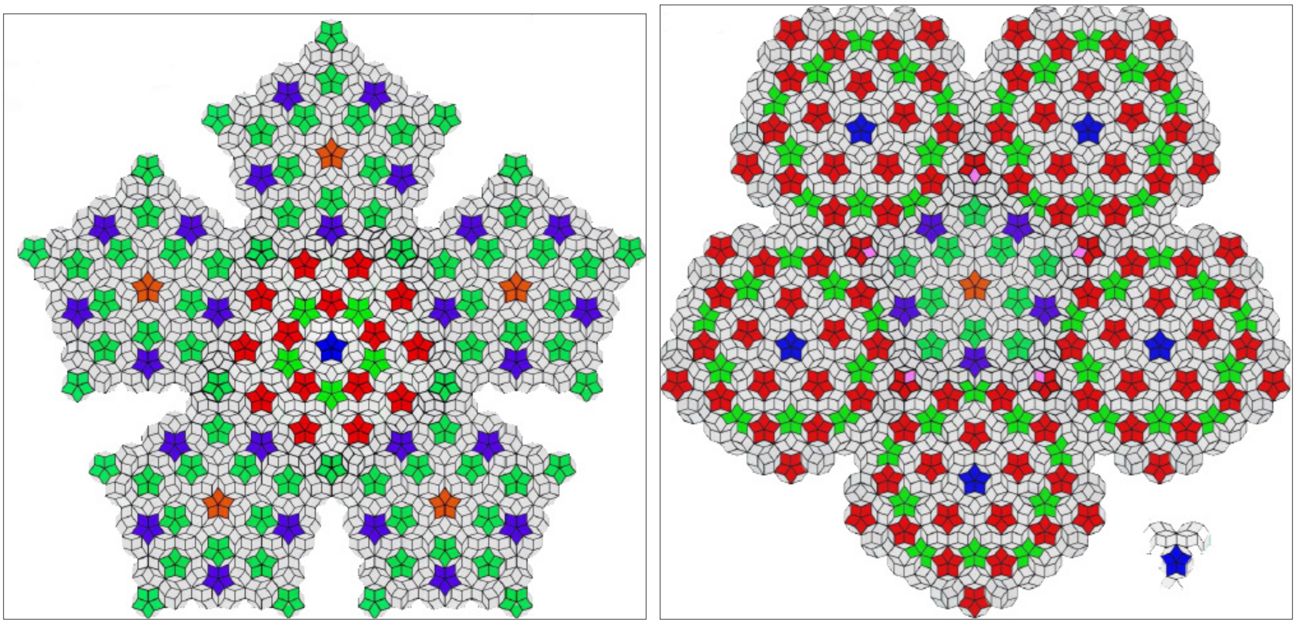


Figure 3 : First run surrounded coupling with unfilled empty area (a) slim type Surrounded couple with swollen type. (b) swollen type surrounded couple with slim type.

It is necessary to establish a standard process for filling the empty space consistency in the latter's larger and infinite empty space, so as to understand how to produce an even larger and still binary tiles. After try error, a cropping technique is introduced here to allow a consistency in filling the empty spaces of larger tiles. Here, only take Fig 3a as example, since it contains much larger empty area remained to be filled, and much more difficulty. By cutting an appropriated area tiles from the top of the fig 3a from type-a decagon down to a position a little below the blue star decagon position, as shown in fig 4a and then stick (patch) them in the blank spaces of Figures 3a. The type -a decagons on the top of cutting areas must be well matched nail-down (embed) into the type-a decagons marked as pink color showing in graph of fig 3a. (as the shown in fig 4b, some potential defective tiles are marked dark grey.as arrow pointed. After the seemingly easy pruning work, suitable tiles, fig 4c are extracted from Figure 4b. There-after, use fig 4c to fill all the five empty areas in Figure 3a, a new Pentagonal binary Penrose slim tile (positive) is completed with 13 decagons on each side as shown fig 5a. The same procedure can be applied to fill the empty space on the fig 3b, a completed pentagonal shaped results are shown in fig 5b which is a bloated tile of 13 decagons each side, but with a negative center.

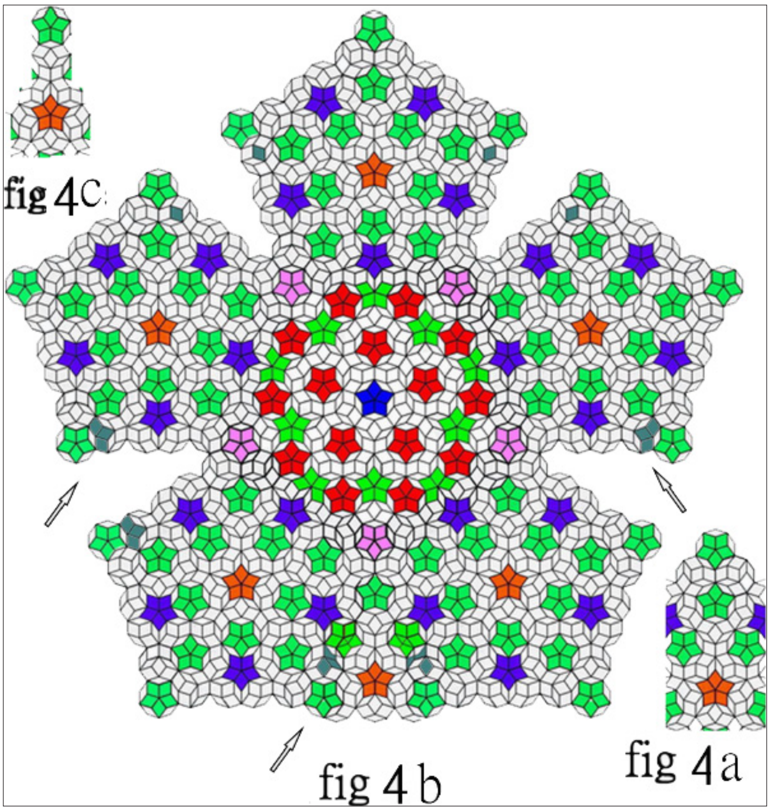
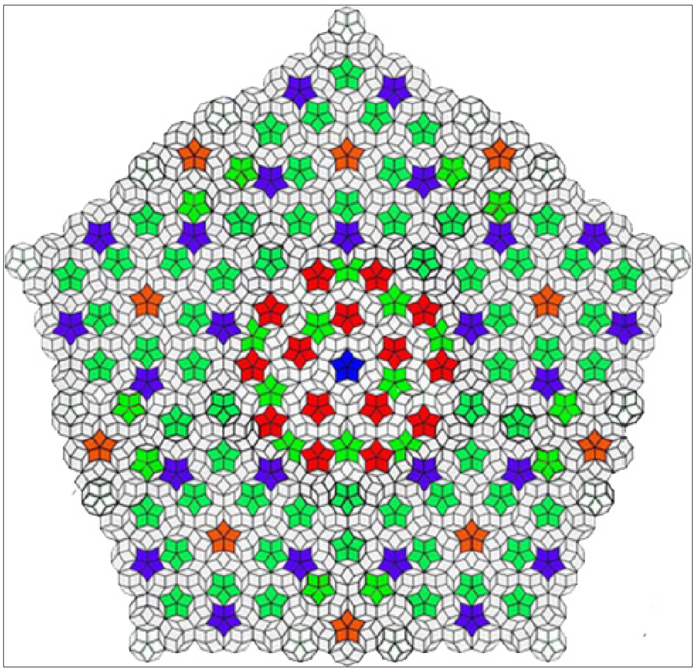


Figure 4: The illustration of filling the empty area, (a) a suggested crop area, (b) result of patched area, and (c) the tile extracted out for filling empty space.



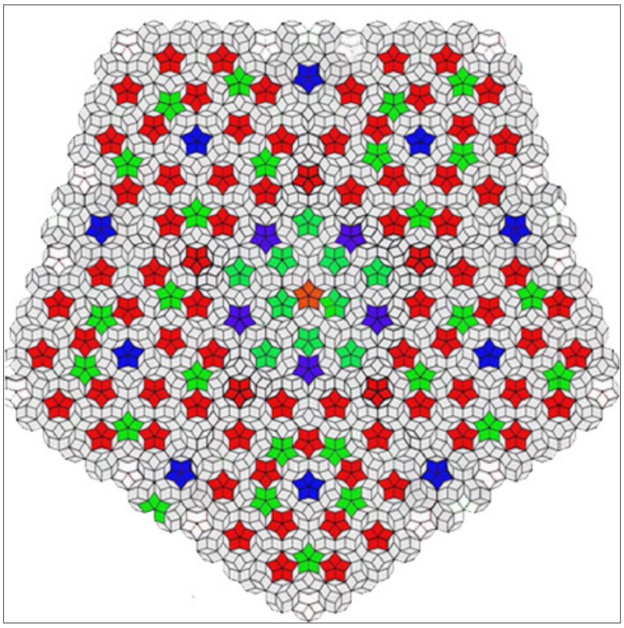


Figure 5 A pair of five-fold symmetry Penrose tiles (in pentagon shape) after first run surrounding couple: (a) slim type positive center, (b) Bloated type negative center, with 13 decagons on each side.

With a new pair of Pentagonal Penrose tiles completed, Penrose tiling of the next (second) run of five surrounding (conjugate) coupling (for fig 5a and fig 5b) are shown in fig 6a and fig 6b., with a 34 decagons each side. And graphs in fig 6 (a, b) are all ready for next third run of surrounding couple. Again, the coupled region of this pair of Penrose tiles and nearby the seam area are able all revised (reversed) to be type-b. A portion of third run coupling result is shown in fig 7, empty area is properly filled , which is a defect-free binary tile.

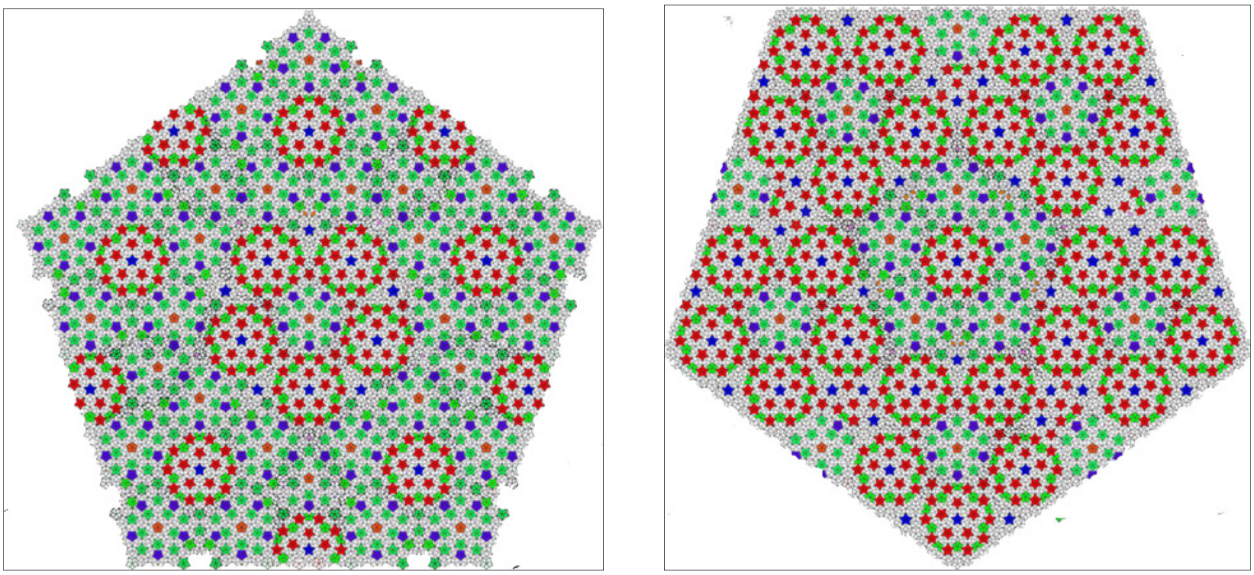


Figure 6: A pair of five-fold symmetry Penrose tiles (in pentagon shape)after second run surrounding couple and empty area filled, with 34 decagons on each side: (a) positive center slim type B) negative center bloated type.

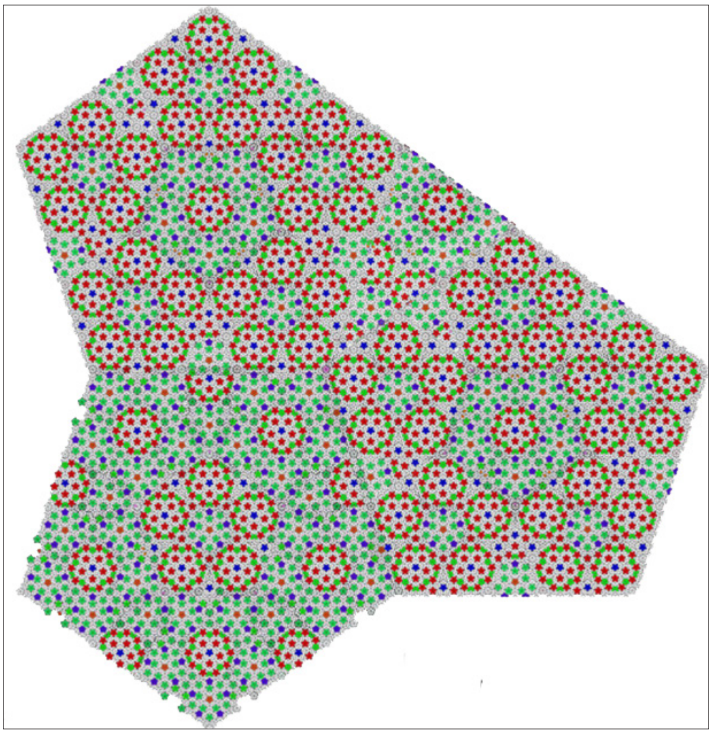


Figure 7: Portion of the tile after the third run conjugated couple with the empty area filled, and seam region defects repaired, a defect free binary tile.

Asymmetric binary tiles can similarly start with different asymmetry bloated shape tiles and different negative tiles. All these tiles have the same decagonal array (profile) on each edge of the tile as the symmetry one, so as the similar empty space. (But the inner of their tiles can be all different). Here, just show some examples portion of the second run coupling result with the empty space filled, as shown in the graph 8 (a) for a positive center bloated type while in figure 8(b) is a slim type, with negative center., which is considered as the s counter part of graph 8(a). The edge of these two tiles can be coupled with a minor revision and can be continued for third run coupling.

Up to this stage, author has confidence to believe that the accomplished of a binary symmetry and asymmetry tiles can be anticipated using the surrounded conjugate coupling.

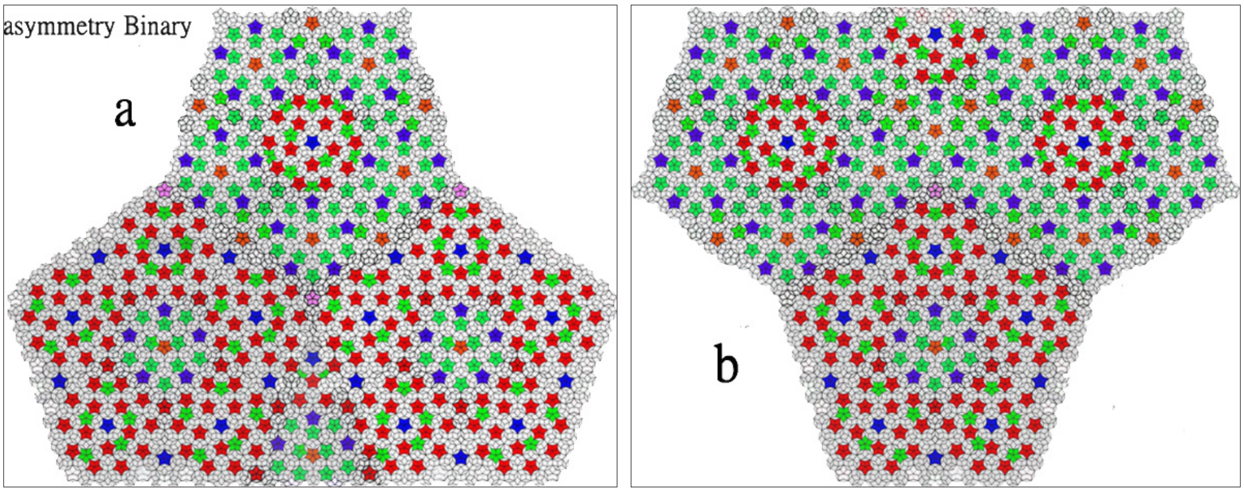


Figure 8: The portion of the defect-free asymmetry binary Penrose tiles after second run of conjugate coupling. (a) positive center (b) negative center.

Summary

In this article, a conjugate coupling method applied on a pair of conjugated binary Penrose tiles is introduced systematically, and extend to a larger binary five-fold symmetry tiles. An asymmetry binary Penrose tiles are also able to be extended by the same procedure. It is anticipated this procedure can be unlimited proceeded and keep the tiles still a binary tiling.

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