Research Article ISSN 2767-3901

International Journal of Mathematic Exploration and Computer Education

Enhancing Solution Techniques for Integro-Differential Equations via the Daftardar-Jafari Method

Prof. Dr. Safwan Aouira[1] & Dr. George Albert Toma[2*]

[1]*Department of Mathematics, Faculty of Science, University of Aleppo, Syria.*

[2]*Department of fundamental sciences, Higher Institute for Applied Sciences and Technology, Aleppo, Syria.*

***Correspondence author**
Dr. George Albert Toma,
Department of fundamental sciences,
Higher Institute for Applied Sciences and Technology,
Aleppo, Syria.

Submitted : 24 Mar 2025 ; Published : 4 Jul 2025

Citation: Toma, G. A. & Aouira, S. (2025). Enhancing Solution Techniques for Integro-Differential Equations via the Daftardar-Jafari Method. . *Int J Math Expl & Comp Edu.2*(2):1-6.

Abstract
This study explores the application of the Daftardar-Jafari Method (DJM) for solving linear and nonlinear integro-differential equations (IDEs). Unlike traditional perturbation-based techniques, DJM does not require small parameters, making it highly effective for strongly nonlinear problems. The method constructs a rapidly converging iterative sequence, providing accurate analytical or approximate solutions with reduced computational complexity. To validate the efficiency and accuracy of DJM, several test problems are presented, demonstrating its superior convergence and computational advantages. The numerical results, obtained using MATLAB, highlight the method's robustness and effectiveness in handling complex IDEs. The findings suggest that DJM is a powerful tool for solving a broad class of integro-differential equations, with potential applications in various scientific and engineering fields.

Keywords: Linear and nonlinear integro-differential equation, Daftardar-Jafari Method, Exact and approximate solution.

Introduction

Integro-differential equations (IDEs) play a crucial role in mathematical modeling across various scientific and engineering disciplines. These equations, which incorporate both differential and integral operators, naturally arise in fields such as fluid mechanics, biological systems, physics, and control theory, where the evolution of a system depends on both its current state and its cumulative history. IDEs are essential tools for describing dynamic systems with memory effects, hereditary properties, and distributed parameters, making their solution a fundamental aspect of advancing both theoretical and applied research. The complexity of solving IDEs arises from their intricate structure, especially when dealing with nonlinear formulations. Obtaining analytical solutions for such equations is often challenging, particularly for real world or highly complex models. Consequently, researchers have explored various analytical and numerical techniques to address these equations. Traditional analytical methods, such as the Laplace transform (Toma, 2022), Fourier transform (Bobolian et al., 2009), and Green's functions (Sawangtong & Sawangtong, 2019), are widely used but are often limited to linear cases. On the other hand, numerical methods, including finite difference methods (Jan et al., 2016), spectral methods, and iterative techniques (Kalouta, 2022; Bassi & Ismail, 2022), provide approximate solutions but may suffer from computational inefficiencies or numerical instability. In recent years, semi-analytical iterative techniques have gained significant attention due to their ability to efficiently handle nonlinear IDEs while maintaining high computational efficiency. Among these methods, the Daftardar-Jafari Method (DJM) has emerged as a powerful and effective approach. Developed by Daftardar-Gejji and Jafari, this iterative technique is designed to generate a rapidly converging series solution for nonlinear differential and integro-differential equations. For example (Batiha & Ghanim, 2021; Patade & Bhalrkar, 2015; Toma, 2021; Toma & Alturky, 2021; Toma & Alturky, 2021), unlike traditional perturbation-based methods (Mhahmood et al., 2024; Gupta et al., 2013; Kharrat & Toma, 2020; Toma, 2021; Toma, & Kharrat, 2021; Toma & Kharrt, 2020), DJM does not require small parameters, making it particularly advantageous for dealing with strongly nonlinear problems. DJM has demonstrated remarkable effectiveness in various applications, successfully solving fractional integro-differential equations, nonlinear boundary value problems, and partial integro-differential equations arising in engineering and physics. The method constructs a sequence of approximate solutions that rapidly converge to the exact or highly accurate solution. Furthermore, DJM reduces the number of computational steps compared to traditional iterative techniques, making it a practical approach for real-world applications. This study aims to expand the investigation into the applicability of DJM for solving both linear and nonlinear integro-differential equations. By leveraging its iterative structure and rapid convergence, we seek to demonstrate its effectiveness in obtaining accurate solutions for a wide range of problems. Additionally, computations are performed using MATLAB software to verify the accuracy and efficiency of the proposed approach. This paper is organized as follows: Section 2 presents the mathematical framework and theoretical foundations of the Daftardar-Jafari Method. Section 3 discusses the application of the method to different types of integro-differential equations, incorporating numerical results and comparative analysis. Finally, Section 4 concludes the study by summarizing key findings and outlining potential future research directions.

Methodology and Formulation

We focus on the following nonlinear integro-differential equation

$$u^{(n)}(x) = f(x) + g\left(x, u, u', \dots, u^{(n-1)}\right) + \int_0^x K(x,t)\, M\big(u(t)\big)dt \tag{1}$$

With initial conditions

$$u^{(k)}(0) = a_k\ , \qquad a_k = const,\ \ k = \overline{0, n-1}$$

Where u is a unknown function, $M(u(t))$ is a linear or nonlinear function, $f(x)$ is a known analytical functions on $[0\ ,b]$ and the kernel is $K(x,t)$.

According to the DJM, by integrating both sides of the equation (1) with respect to x

$$u(x) = \underbrace{\int_0^x \dots \int_0^x}_{n} f(x)\, \underbrace{dx \dots dx}_{n} + \underbrace{\int_0^x \dots \int_0^x}_{n} g\left(x, u, u', \dots, u^{(n-1)}\right) \underbrace{dx \dots dx}_{n} + \underbrace{\int_0^x \dots \int_0^x}_{n} \int_0^x K(x,t)\, M\big(u(t)\big)dt\ \underbrace{dx \dots dx}_{n} \tag{2}$$

Where the nonlinear part

$$N(u) = \underbrace{\int_0^x \dots \int_0^x}_{n} g\left(x, u, u', \dots, u^{(n-1)}\right) \underbrace{dx \dots dx}_{n} + \underbrace{\int_0^x \dots \int_0^x}_{n} \int_0^x K(x,t)\, M\big(u(t)\big)dt\ \underbrace{dx \dots dx}_{n}$$

The solution to the equation (2) is given in the form

$$\boldsymbol{u} = \sum_{(i=0)}^{\infty} \boldsymbol{u_i} \tag{3}$$

By expressing the nonlinear part in the form

$$N\left(\sum_{i=0}^{\infty} u_i\right) = \underbrace{\int_0^x \dots \int_0^x}_{n} g\left(x, \sum_{i=0}^{\infty} u_i, \sum_{i=0}^{\infty} u'_i, \dots, \sum_{i=0}^{\infty} u_i^{(n-1)}\right) \underbrace{dx \dots dx}_{n} + \underbrace{\int_0^x \dots \int_0^x}_{n} \int_0^x K(x,t)\, M\left(\textstyle\sum_{i=0}^{\infty} u_i(t)\right)dt\ \underbrace{dx \dots dx}_{n} \tag{4}$$

By substituting (3) and (4) into (2), we obtain:

$$u(x) = \underbrace{\int_0^x \dots \int_0^x}_{n} f(x)\, \underbrace{dx \dots dx}_{n} + N\left(\sum_{i=0}^{\infty} u_i\right)$$

From this, the following results:

$$u_0 = \underbrace{\int_0^x \dots \int_0^x}_{n} f(x)\, \underbrace{dx \dots dx}_{n}$$

$$\boldsymbol{u_1 = N(u_0)}$$

$$\boldsymbol{u_2 = N(u_0 + u_1) - N(u_0)}$$

$$u_m = N(u_0 + u_1 + \dots + u_{m-1}) - N(u_0 + u_1 + \dots + u_{m-2})\ ,\ m = 2,3,\dots$$

For convergence of the DJM

Lemma (Bhalekar & Daftardar-Gejji, 2011). If N is C^{∞} in a neighborhood of u_0 and $\|N^{(n)}(u_0)\| < L$, for any n and for some real $L > 0$ and $\|u_i\| \le M < e^{-1}$, $i = 1,2,\dots$ then the series $\sum_{n=0}^{\infty} u_n$ is absolutely convergent and $\|u_i\| \le LM^n e^{n-1}(e-1)$, $n=1,2,\dots$

Test Problems

Example 1

Consider the following linear integro-differential equation

$$u'(x) = 1 + \int_0^x u(t)\, dt\ ,\quad 0 \le x, t \le 1 \tag{5}$$

With the initial condition

$$u(0)=1$$

According to the DJM, by integrating both sides of the equation (5) with respect to x.

$$u(x) = x + \int_0^x \int_0^x u(t)\, dt\, dx + c$$

By using $u(0)=1$, yields

$$u(x) = x + 1 + \int_0^x \int_0^x u(t)\, dt\, dx$$

Where

$$f(x) = u_0(x) = 1 + x\ , \qquad N(x) = \int_0^x \int_0^x u(t)\, dt\, dx$$

Then

$$u_1(x) = N(u_0) = \int_0^x \int_0^x u_0(t)\, dt\, dx = \frac{x^2}{2} + \frac{x^3}{6}$$

$$u_2(x) = N(u_0 + u_1) - N(u_0) = \frac{x^4}{24} + \frac{x^5}{120}$$

$$u_2(x) = N(u_0 + u_1 + u_2) - N(u_0 + u_1) = \frac{x^6}{720} + \frac{x^7}{5040}$$

Then the exact solution is

$$u(x) = \sum_{i=0}^{\infty} u_i = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \dots = e^x$$

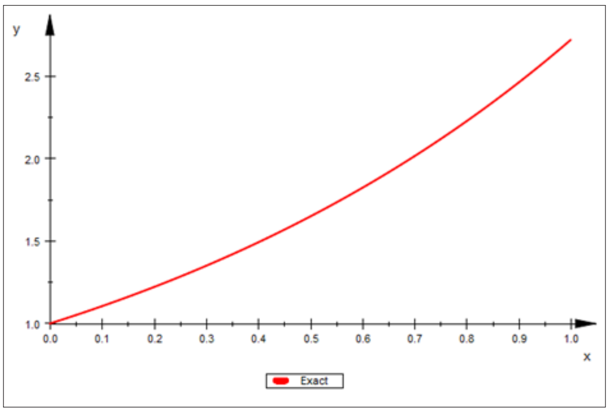

Figure1: Exact solution for example 1

Example 2

Consider the following linear integro-differential equation

$$u^{(3)}(x) - x\,u''(x) = \frac{4}{7}x^9 - \frac{8}{5}x^7 - x^6 + 6x^2 - 6 + 4\int_0^x x^2\,t^3\,u(t)\;dt\,,\quad 0 \le x\,,t \le 1 \tag{6}$$

with the initial conditions

$$u(0) = 1\;\;,\;\; u'(0) = 2\quad,\quad u''(0) = 0$$

According to the DJM, by integrating both sides of the equation (6) with respect to x

$$u(x) = \frac{x^{12}}{2310} - \frac{x^{10}}{450} - \frac{x^9}{504} + \frac{x^5}{10} - x^3 + \int_0^x\int_0^x\int_0^x x\,u''(t)\,dx\,dx\,dx + 4\int_0^x\int_0^x\int_0^x\int_0^x x^2\,t^3\,u(t)\,dt\,dx\,dx\,dx + c$$

By using $u(0) = 1$, $u'(0) = 2$, $u''(0) = 0$ yields

$$u(x) = \frac{x^{12}}{2310} - \frac{x^{10}}{450} - \frac{x^9}{504} + \frac{x^5}{10} - x^3 + 2x + 1 + \int_0^x\int_0^x\int_0^x x\,u''(t)\,dx\,dx\,dx + 4\int_0^x\int_0^x\int_0^x\int_0^x x^2\,t^3\,u(t)\,dt\,dx\,dx\,dx$$

Where

$$f(x) = u_0(x) = \frac{x^{12}}{2310} - \frac{x^{10}}{450} - \frac{x^9}{504} + \frac{x^5}{10} - x^3 + 2x + 1$$

$$N(x) = \int_0^x\int_0^x\int_0^x x\,u''(t)\,dx\,dx\,dx + 4\int_0^x\int_0^x\int_0^x\int_0^x x^2\,t^3\,u(t)\,dt\,dx\,dx\,dx$$

Then

$$\begin{aligned} u_1(x) = N(u_0) &= \int_0^x\int_0^x\int_0^x x\,u_0''(t)\,dx\,dx\,dx + 4\int_0^x\int_0^x\int_0^x\int_0^x x^2\,t^3\,u_0(t)\,dt\,dx\,dx\,dx \\ &= -\frac{x^5}{10} + \frac{x^7}{105} + \frac{x^9}{504} + \frac{x^{10}}{450} - \frac{x^{11}}{6930} - \frac{9x^{12}}{15400} + \frac{4x^{14}}{85995} - \frac{x^{18}}{8019648} - \frac{x^{19}}{9157050} \\ &\quad + \frac{x^{21}}{73735200} \end{aligned}$$

$$\begin{aligned} u_2(x) &= N(u_0 + u_1) - N(u_0) \\ &= -\frac{x^7}{105} + \frac{x^9}{1260} + \frac{x^{11}}{6930} + \frac{x^{12}}{6600} - \frac{x^{13}}{108108} - \frac{383x^{14}}{6879600} + \frac{31x^{16}}{8731800} + \frac{x^{18}}{8019648} \\ &\quad + \frac{x^{19}}{9157050} - \frac{1657x^{20}}{147891744000} - \frac{13x^{21}}{565488000} + \frac{20747x^{23}}{13750604627760} - \frac{x^{27}}{774096523200} \\ &\quad + \frac{x^{28}}{1034948105100} + \frac{x^{30}}{11226184200000} \end{aligned}$$

Then the approximate solution is

$$u(x) = \sum_{i=0}^{\infty} u_i = u_0 + u_1 + u_2 + \cdots$$

Table 1 shows the absolute errors at different values of x for two terms $u_0 + u_1$ and three terms $u_0 + u_1 + u_2$ of the DJM. Where the exact solution of Eq. (6) is $u(x) = 1 + 2x - x^3$.

x	Error of DJM $u_0 + u_1$ $n = 2$	Error of DJM $u_0 + u_1 + u_2$ $n = 3$
0	0	0
0.1	9.52379×10^{-10}	7.93650×10^{-13}
0.2	1.21901×10^{-7}	4.06341×10^{-10}
0.3	2.08252×10^{-6}	1.56195×10^{-8}
0.4	7.43002×10^{-5}	2.07966×10^{-7}
0.5	7.43002×10^{-5}	1.54847×10^{-6}
0.6	2.65789×10^{-4}	7.97992×10^{-6}
0.7	7.79691×10^{-4}	3.18867×10^{-5}
0.8	1.97652×10^{-3}	1.05710×10^{-4}
0.9	4.47774×10^{-3}	3.03686×10^{-4}
1.0	9.27429×10^{-5}	7.78774×10^{-4}

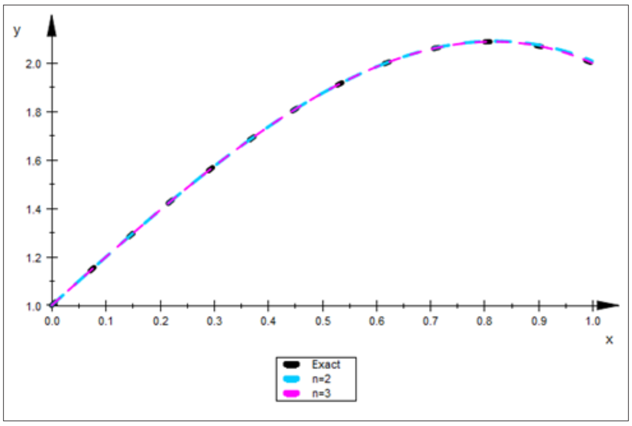

Figure 2: Comparison of exact and approximate solution for example 2

Example 3

Let us consider the following nonlinear integro-differential equation

$$u^{(4)}(x) = e^{-3x} + e^{-x} - 1 + 3\int_0^x u^3(t)\,dt \quad , \quad 0 \le x,t \le 1 \qquad (7)$$

With the initial conditions

$u(0) = u''(0) = 1, \quad u'(0) = u^{(3)}(0) = -1$

According to the DJM, by integrating both sides of the equation (7) with respect to x and by using $u(0) = u''(0) = 1$, $u'(0) = u^{(3)}(0) = -1$ yields

$$u(x) = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \frac{x^5}{30} + \frac{x^6}{72} - \frac{x^7}{180} + \frac{41x^8}{20160} - \frac{61x^9}{90720} + \frac{73x^{10}}{362880} - \frac{547x^{11}}{9979200} + \frac{3281x^{12}}{239500800} - \frac{703x^{13}}{222393600} + \frac{1181x^{14}}{743565824} + 3\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x u^3(t)\,dt\,dx\,dx\,dx\,dx$$

Where

$$f(x) = u_0(x) = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \frac{x^5}{30} + \frac{x^6}{72} - \frac{x^7}{180} + \frac{41x^8}{20160} - \frac{61x^9}{90720} + \frac{73x^{10}}{362880} - \frac{547x^{11}}{9979200} + \frac{3281x^{12}}{239500800} - \frac{703x^{13}}{222393600} + \frac{1181x^{14}}{743565824}$$

$$N(x) = 3\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x u^3(t)\,dt\,dx\,dx\,dx\,dx$$

Then

$$u_1(x) = N(u_0) = 3\int_0^x\int_0^x\int_0^x\int_0^x\int_0^x u_0^3(t)\,dt\,dx\,dx\,dx\,dx$$
$$= \frac{x^5}{40} - \frac{x^6}{80} + \frac{3x^7}{560} - \frac{9x^8}{4480} + \frac{3x^9}{4480} - \frac{x^{10}}{4800} + \frac{x^{11}}{15400} - \frac{3x^{12}}{140800} + \frac{421x^{13}}{57657600} - \frac{449x^{14}}{179379200} + \frac{27x^{15}}{32032000} - \frac{7243x^{16}}{25830604800} + \frac{8497x^{17}}{91483392000} - \frac{2693x^{18}}{8782405320} + \frac{22616071x^{19}}{2252687044608000} - \frac{220339307x^{20}}{67580611338240000} + \frac{30958261x^{21}}{29566517460480000} - \frac{414593x^{22}}{1249389453312000} + \frac{109508713x^{23}}{1056046435411968000} + \cdots$$

Then the approximate solution is

$$u(x) = \sum_{i=0}^{\infty} u_i = u_0 + u_1 + u_2 + \cdots$$

Table 2 shows the absolute errors at different values of x for two terms $u_0 + u_1$ of the DJM.

Where the exact solution of (7) is $u(x) = e^{-x}$.

x	Error of DJM $u_0 + u_1$
0	0
0.1	1.0×10^{-10}
0.2	1.0×10^{-10}
0.3	1.0×10^{-10}
0.4	4.0×10^{-10}
0.5	3.8×10^{-9}
0.6	2.1×10^{-8}
0.7	8.4×10^{-8}
0.8	2.8×10^{-7}
0.9	7.9×10^{-7}
1.0	2.0×10^{-6}

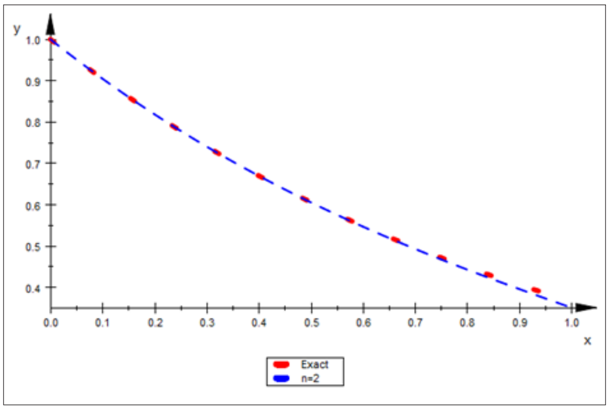

Figure 3: Comparison of exact and approximate solution for example 3

Example 4

Let us consider the following nonlinear integro-differential equation

$$u'(x) = -1 + \int_0^x u^2(t)\,dt \quad , \quad 0 \le x,t \le 1 \tag{8}$$

With the initial condition

$$u(0) = 0$$

According to the DJM, by integrating both sides of the equation (8) with respect to x and by using $u(0) = 0$ yields

$$u(x) = -x + \int_0^x \int_0^x u^2(t)\,dt\,dx$$

Where

$$f(x) = u_0(x) = -x \quad \& \quad N(x) = \int_0^x \int_0^x u^2(t)\,dt\,dx$$

Then

$$u_1(x) = N(u_0) = \int_0^x \int_0^x u_0^2(t)\,dt\,dx = \frac{x^4}{12}$$

$$u_2(x) = -\frac{x^7}{252} + \frac{x^{10}}{12960}$$

$$u_3(x) = \frac{x^{10}}{11340} - \frac{37x^{13}}{7076160} + \frac{109x^{16}}{914457600} - \frac{x^{19}}{558472320} + \frac{x^{22}}{77598259200}$$

Table 3: shows the absolute errors at different values of x for two terms $u_0 + u_1$ and three terms $u_0 + u_1 + u_2$ and four terms $u_0 + u_1 + u_2 + u_3$ of the DJM.

x	Error of DJM $u_0 + u_1$	Error of DJM $u_0 + u_1 + u_2$	Error of DJM $u_0 + u_1 + u_2 + u_3$
0	0	0	0
0.1	2.7777×10^{-8}	8.8177×10^{-18}	1.4696×10^{-17}
0.2	1.7774×10^{-6}	4.5122×10^{-10}	6.0145×10^{-14}
0.3	2.0223×10^{-5}	1.7321×10^{-8}	7.7867×10^{-12}
0.4	1.1358×10^{-4}	2.3003×10^{-7}	2.4478×10^{-10}
0.5	4.3252×10^{-4}	1.7058×10^{-6}	3.5374×10^{-9}
0.6	1.2882×10^{-3}	8.7398×10^{-6}	3.1214×10^{-8}
0.7	3.2369×10^{-3}	3.4653×10^{-5}	1.9563×10^{-7}
0.8	7.1782×10^{-3}	1.1375×10^{-4}	9.5276×10^{-7}
0.9	1.4463×10^{-2}	3.2283×10^{-4}	3.8203×10^{-6}
1.0	2.7006×10^{-2}	8.1573×10^{-4}	1.3117×10^{-5}

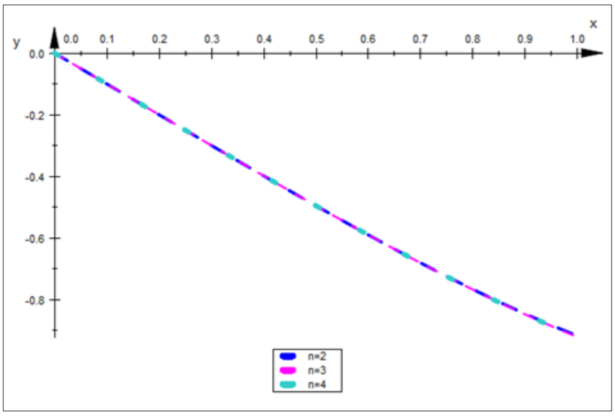

Figure 4: Comparison of approximate solution for example 4

Conclusion and Discussion

In this study, the Daftardar-Jafari Method (DJM) was applied to solve linear and nonlinear integro-differential equations. The results demonstrate that DJM provides accurate solutions efficiently, reducing computational complexity compared to traditional methods. By generating a rapidly converging series, the method ensures high precision with fewer iterative steps. The effectiveness of DJM has been validated through several test problems, showing its reliability in solving complex IDEs. Given its simplicity, rapid convergence, and broad applicability, DJM proves to be a powerful tool for tackling integro-differential equations in scientific and engineering fields. Future research could focus on extending DJM to more complex systems and analyzing its convergence properties in greater detail.

References

1. Toma, G. A. (2022). Exact Solution for Neutrosophic System of Ordinary Differential Equations by Neutrosophic Thick Function Using Laplace Transform. *International Journal of Neutrosophic Science (IJNS), 19*(1), 212-216. DOI: http://dx.doi.org/10.54216/IJNS.190116
2. Bobolian, E., Saedian, J. & Paripour, M. (2009). Computing the Fourier Transform via Homotopy Perturbation Method. Z. Naturforsch, 64a(11), 671 – 675. DOI: http://dx.doi.org/10.1515/zna-2009-1101
3. Sawangtong, P. & Sawangtong, W. (2019). Green's function homotopy perturbation method for the initial-boundary value problems. *Advanced in Difference Equations, 2019*(1). DOI: https://advancesindifferenceequations.springeropen.com/articles/10.1186/s13662-019-2350-7
4. Jan, B., Michal, H., Juha, H. & Tomas, L. (2016). Numerical Solution of nonlinear integro-differential equations. The European *Physical Journal Conferences*, 108, 02017. DOI: http://dx.doi.org/10.1051/epjconf/201610802017
5. Kalouta. A. (2022). A novel iterative method to solve nonlinear wave-like equations of fractional order with variable coefficient. *Revista Colombiana de Matem ́aticas, 56*(1), 13-34. DOI: http://dx.doi.org/10.15446/recolma.v56n1.105612
6. Bassi, N. & Ismail, A. (2022). The Modified Variational Iteration Method to Solve Linear Fractional Differential Equations. *Journal of Advances in Mathematics and Computer Science, 37*(8), 57-64. DOI: https://doi.org/10.9734/jamcs/2022/v37i830472
7. Batiha, B. & Ghanim, F. (2021). Numerical Implementation of Daftardar-Gejji and Jafari Method to the Quadratic Riccati Equation. Buletnul. Academiel. De s¸Tiint¸ E a Republicii Moldova. *Mathematica, 3*(97), 21–29. https://www.math.md/files/basm/y2021-n3/y2021-n3-(pp21-29).pdf4
8. Patade, J. & Bhalrkar, S. (2015). A New Numerical Method Based onDaftardar-Gejji and Jafari Technique for Solving Differential Equations. *World Journal of Modelling and Simulation, 11*(4), 256-271. https://www.researchgate.net/publication/283498636_A_new_numerical_method_based_on_Daftardar-Gejji_and_Jafari_Technique_for_solving_differential_equations
9. Toma, G. A. (2021). Solving System of Integro-Differential Equations Using a New Hybrid Semi-Analytical Method. *Journal of Mathematics Sciences & Computational Mathematics, 3*(1), 66-77. DOI: http://dx.doi.org/10.15864/jmscm.3105
10. Toma, G. A. & Alturky, S. (2021). Applying a New Hybrid Approach to Provide Exact Solutions for Partial Integro-Differential Equations. *Journal of Mathematics Sciences & Computational Mathematics, 2*(4), 524-531. DOI: http://dx.doi.org/10.15864/jmscm.2406
11. Toma, G. A. & Alturky, S. (2021). A Hybrid Kharrat-Toma Transform with Homotopy Perturbation Method for Solving Integro-Differential Equations. *Journal of Mathematical Analysis and Modeling, 2*(2), 50–62. https://sabapub.com/index.php/jmam/article/view/233
12. Mhahmood, R. F., Hama, S. S, Fateh, S. S. & Hilmi, H. (2024). Using Kharrat-Toma transform to solve a growth and decay problem. *Tikrit Journal of Pure Science, 29*(6), 70-79. DOI: http://dx.doi.org/10.25130/tjps.v29i6.1685
13. Gupta, S., Kuma, D. & Singh, J. (2013). Application of He's homotopy perturbation method for solving nonlinear wave-like equations with variable coefficients. *International Journal of Advances in Applied Mathematics and Mechanics, 1*(2), 65-79. http://www.ijaamm.com/uploads/2/1/4/8/21481830/ijaamm-y2013v1n2p6-devendrakumar.pdf
14. Kharrat, B. N. & Toma. G. A. (2020). A New Hybrid Sumudu Transform With Homotopy Perturbation Method For Solving Boundary Value Problems. *Middle-East Journal of Scientific Research, 28*(2), 142-149. DOI: http://dx.doi.org/10.5829/idosi.mejsr.2020.
15. Toma, G. A. (2021). Application of ZZ-Transform with Homotopy Perturbation Method for Solving Higher-Order Boundary Value Problems. *Journal of Mathematics Sciences & Computational Mathematics, 3*(1), 134-141. DOI: http://dx.doi.org/10.15864/jmscm.3111
16. Toma, G. A. & Kharrat, B. N. (2021). Combination of Kharrat-Toma Transform and Homotopy Perturbation Method to Solve a Strongly Nonlinear Oscillators Equation. *Journal of Sciences, Computing and Engineering Research, 2*(1), 143-146. DOI: https://doi.org/10.46379/jscer.2021.020102
17. Toma, G. A. & Kharrt. B. N. (2020). A Hybrid Homotopy Perturbation Method with Natural Transform to Solve Partial Differential equations. *International Journal of Scientific Research in Mathematical and Statistical Sciences, 7*(4), pp.14-19. https://www.researchgate.net/publication/344264347_A_Hybrid_Homotopy_Perturbation_Method_with_Natural_Transform_to_Solve_Partial_Differential_equations
18. Bhalekar, S. & Daftardar-Gejji, V. (2011). Convergence of the New Iterative Method. *International Journal of Differential Equations, 2011*(10), 989065. DOI: http://dx.doi.org/10.1155/2011/989065.