Research Article ISSN 2766-2624

Advances in Earth and Environmental Science

Towards A Time-Integrated Convolution Of The Fractal Biodiversity Interface : The Relation Between Biodiversity and Entropy Re-examined

Wilfried Allaerts

Biological Publishing A&O and Erasmus MC, Rotterdam, The Netherlands.

***Corresponding author**

Wilfried Allaerts,
Biological Publishing A&O and Erasmus MC,
Rotterdam,
The Netherlands.

Submitted : 14 May 2025 ; Published : 9 Jul 2025

Citation: W Allaerts., (2025). Towards A Time-Integrated Convolution of the Fractal Biodiversity Interface: The Relation Between Biodiversity and Entropy Re-Examined. *Adv Earth & Env Sci; 6*(3):1-9. DOI : https://doi.org/10.47485/2766-2624.1071

Abstract

This paper starts from two premises, namely at first, Erwin Schrödinger's Cat metaphor, as an access to the theory of quantum physics and, secondly, Jacob D. Bekenstein's theory of the entropy of a black hole. Following Erik Verlinde's interpretation of the relationship between entropy and Bekenstein's black hole surface area notion, we further analyze a possible relationship between the entropy or negentropy notion and the biodiversity function of a fractal border zone between ecosystems. Ecosystem borders or interfaces have been discovered as important drivers of biodiversity resilience as well as of biodiversity impairment. The potential use of several mathematical techniques, in particular the time-integrated convolution using Laplace transformation of the biodiversity function, as well as combinatorics and probabilistic network theory are discussed. For, after all, biodiversity estimation is nothing similar to counting animals in a zoo, nor bees in a jar.

Keywords: Schrödinger's Cat Metaphor, Relationship Entropy-Area, Fractal Ecosystem Border-Interface, Laplace Transform Convolution, Combinatorics, Probabilistic Scale-free Networks.

Introduction

The strange correlation between entropy, quantum physics and live and dead of an animal (notably a mammal pet) has a serious historical origin, well known as Schrödinger's Cat metaphor (1). Erwin Schrödinger (1887-1961) suggested his famous thought experiment in 1935, as a reaction to the so-called Copenhagen interpretation of quantum physics. Invisible from outside, being enclosed inside a sealed box (or house) with a Geiger counter (measuring the decay of a radioactive substance) and a bit of poison (hydrocyanic acid released from a shattered flask by a hammer, activated by the Geiger counter), quantum mechanics says that after a while, the cat is both alive and dead (1), (2). When someone opens the box or looks into the house, the cat will be either found alive or dead; however, before that moment, the cat's situation is assumed to be indeterminate. Schrödinger expressed this correlation as follows: the "indeterminacy originally restricted to the atomic domain became transformed into a macroscopic indeterminacy (which can be resolved by direct observation}" (2).

The indeterminacy correlation between the microscopic (atomic state) and the macroscopic scales, as described in Schrödinger's Cat (thought) experiment is also reflected in the conjecture of Jacob D. Bekenstein (1947-2015), stating that black holes should have an entropy and that the entropy of a black hole is proportional to its area (3). To Bekenstein we owe the notion of the relation between area and the amount of information stored (following the concept of a holographic area, according to E. Verlinde) (4), although, according to Bekenstein, due attention should be given to the caveat that a black hole's mass cannot be below the Planck mass ($\approx 2{,}176\ 10^{-8}$ kg), referring to the work of Max Planck (1858-1947). Because if it were, "the black hole would then be smaller than its own Compton length, and would therefore not exhibit the black hole hallmark, the event horizon" (5). On the important role of the length in this analysis, as Bekenstein's referring to the work of A.H. Compton (1892-1962), we elaborate in a paragraph below. However, in order to compare the notion of entropy with the macroscopic state of a system of biological organisms and ecosystems, we will have to explore the origins of Ludwig E. Boltzmann's (1844-1906) thoughts on entropy as a measure of the probabilities of successive states of a system (see **Entropy: definitions and relation between area and amount of information**).

In the present paper, in analogy with Bekenstein's argument (3), (5) on the indeterminacy of macroscopic states, biological estimates such as the biodiversity indices (6) are linked with the notions of entropy change and information (loss). Also here, the concept of interface area is pivotal, making use of the fractal nature of biodiversity systems that depend on the interface between areas for breeding, foraging and resting behavior of living animals (7).

Because the estimation of biodiversity is especially important in relation to situations where the planet's biodiversity is challenged, such as due to the effects of anthropogenic interference, climate change effects or situations where a spontaneous natural biodiversity resilience occurs are at stake. In the fractal approach of biodiversity estimation, however, a particular problem results from the discontinuous character of the fractal mathematics, being represented by geometrically non-differentiable functions (7). A special paragraph therefore is devoted to the use of Laplace transforms in convolution of discontinuous functions (see **Time-integrated convolution of fractal functions using Laplace transformation**). However, the indeterminacy of complex ecosystems remains a challenge to deterministic modeling. Or, in other words, old 'demons' re-appear when a conceptual link between entropy and macroscopic systems is envisaged.

The question of the entropy of biodiversity changes is related to important practical questions like the notion of a critical dimensionality, or the percolation probability, which also may be an important characteristic for an ecosystem's resilience or robustness (6) (see **Implications for biodiversity resilience and ecosystem robustness**). Finally, also alternative approaches, taking benefit of important developments in combinatorial mathematics and probabilistic methods, are critically examined (see **Possible Applications of Combinatorial Mathematics in Ecology**).

Entropy: definitions and relation between area and amount of information

According to Schrödinger (1967), the original formula of entropy results from Boltzmann's works on *Gas theory* (8) and on *Entropy and Probability Theory* (9). Boltzmann's equation of entropy is known as:

$$-\,entropy = k \log\left(\frac{1}{D}\right)$$

D, thus being a direct measure of order (1). The negative sign of entropy is also found in the formulation of 'negentropy' by Léon Brillouin (1889-1969) (10):

$$\mathbf{H= -\, \Sigma}\, \boldsymbol{p(i)}\ \mathbf{log}\ \boldsymbol{p_i}$$

Another way of expressing the same relationship, which indeed is the expression literally carved out in stone on Boltzmann's tombstone, namely:

$$\boldsymbol{S = k\ log\ W}$$

With *S* for entropy, *k* the so-called Botzmann's constant and *W* (from the German 'Wahrscheinlichkeit') for the probability, which is also interpreted as the degree of dis-order of the system. For Boltzmann it was evident that the Second Law of Thermodynamics, stating that transformations in a closed system always resulted in an increase in entropy, was just another way of stating that the transformations in a closed system would always tend to a state that is more probable than the previous state (11). Presumably, it was added in his seminal works, without knowing the exact, original state of the system, but nevertheless, giving an outcome for the comparison of successive states. We will resume this matter of increased probability of successive states later on, when referring to the extrapolation of that notion of probability of biodiversity impairment, resilience or anthropogenic influences. Boltzmann's original theory was developed for atomic particles, giving rise to his kinetic gas theory too (8), but his theory could also become applied to polyatomic molecules (11).

Bekenstein's analysis, on the other hand, originated from the difficult questions that were raised, following the acknowledgment of the disappearance of information, when an object as big as a star or even a galaxy disappears into a black hole (12). Bekenstein formulated the relation between area and amount of information. He established that a minimal change of entropy (Δ S), for a particle entering a black hole, could be defined as follows (3):

$$\Delta S = \frac{h}{\pi}.\, df/d\alpha = \ln 2$$

After integration of *f*(*α*) for the rationalized surface of the black hole with entropy S_{bh} , Bekenstein obtained (3):

$$\boldsymbol{S_{bh} = (1/2\ ln\ 2/4\ \pi)\ k_B\ c^{-3}\ 2\pi\ h^{-1}\ G^{-1}\ A}$$

with k_B for Boltzmann's constant (~1, 38.. 10^{-23} JK^{-1}), *h* the Planck constant (≈ 6,626.. 10^{-34} Js)(related to Planck's length) (see below), c the speed of light (≈ 2,9979.. 10^{8} ms^{-1}), *G* the gravitation constant and A the surface of the black hole (3). Also in E. Verlinde's work on the entropic nature of gravity (13), following the argumentation of Bekenstein, it is argued that the change of entropy (Δ*S*) was linear in terms of the displacement (Δ*x*) (towards a holographic screen, in analogy to Bekenstein's horizon of a black hole). The argument of linearity is directly derived from the definition of the Compton wavelength for a particle with (rest) mass m:

$$\boldsymbol{\lambda=h/mc}$$

The Compton wavelength of an electron (with 'rest' mass ≈ 9,109.. 10^{-31} kg) thus approaches 2,4263.. 10^{-12} m. For a 'heavy' object like an animal (or, the metaphor alike, for Schrödinger's cat) this *m* is approximately 10^{30} times higher. This implicates that λ becomes 10^{30} times smaller, approximating 10^{-42} m, which is far below the smallest possible length, called the Planck length (l_p) (≈ 1,616.. 10^{-35} m), or, the application of the above entropy argument to macroscopic objects is physically meaningless.

However, Bekenstein already argued that, for instance in applying entropy to astronomic black holes, the notion of entropy is directly related to the event horizon property (resulting from the gravitation of a gigantic mass) of the black hole (see. **Introduction**). According to Verlinde(4), Bekenstein discovered that there is a relation between area (of an event horizon) and the amount of information stored. Verlinde explains: “Then you should ask yourself: What is information? According to Boltzmann, entropy is defined as the logarithm of the number of states. In his epoch, this was an interesting point of view, but I would rather reverse the way of thinking. I would rather think of an area as something which is defined through the holographic principle” (Allaerts & Vellekoop, 2012). For Verlinde, area is something dividing one part of space from another, which is also a way of looking at information: “in quantum mechanics, it is called entanglement!” (4).

Alternatively, instead of trying to assign a quantum mechanical correlate for the probabilistic contribution of an animal to the entropy of a system (such as in Schrödinger’s metaphor) (see **Introduction**), one may link the animal to its presence or absence in a certain territory, or to the biodiversity of that ecosystem, when summed over all the life forms present (or absent) in that area. Also here, size does matter, because an animal’s territory in some way is (directly) related to its size or mass, or, (indirectly) through the number of offspring or reproductive behavior (7) (see below).

Time-integrated convolution of fractal functions using Laplace transformation

As it has been known for long time, the use of P.S. Laplace’s (1749-1827) transformation in mathematical analysis may have some distinct advantages (14), such as that it can be applied to functions that are (only) piecewise continuous, as well as to all operational rules like addition, multiplication, differentiation and integration (14). The Laplace Transform of a given function $F(t)$ that is defined for all real $t > 0$ or $t = 0$ as the integral:

$$\int_0^\infty e^{-st}F(t)dt = lim_{\epsilon\to 0+} lim_{T\to\infty} \int_\epsilon^T e^{-st}F(t)dt$$

$$\mathcal{L}\{F(t)\} = \int_0^\infty e^{-st}F(t)dt = f(s)$$

An interesting property results from the Laplace transform of the error function of t ($erf\ t$) and its complement ($erfc\ t$) (see below). Error functions and their complements are known to result from Laplace transforms (and their inverses) of functions encountered in reaction-diffusion theory, such as Bessel functions (15). The error function terms represent the stochastic nature of these phenomena:

$$erf\ t = 2\pi^{-\frac{1}{2}} \int_0^t e^{-x^2} dx$$

$$erfc\ t = 2\pi^{-\frac{1}{2}} \int_t^\infty e^{-x^2} dx$$

$$erfc\ t = 1 - erf\ t$$

$$\mathcal{L}\{erfc\ t^{\frac{1}{2}}\} = \mathcal{L}\{1\} - \mathcal{L}\{erf\ t^{\frac{1}{2}}\}$$

Biodiversity entails the notion of adding up the occurrences of distinct species (in a certain area). In order to find a suitable function to describe the local biodiversity of a system, we may take advantage of the following relations. First, the notion of ‘overall survival prognosis’ ($P_{overall}$), defined in terms of a ‘Density of Information Processing’ (DIP) (16), also known as ‘trajectory density’ (17), in simplified form (18) given as:

$$\boldsymbol{P_{overall}\ (x_{age},\ t) = 1 - \Sigma_{i,j} N \int^{\infty}_{t=0} DIP\ (x, i.t)\ (1 - e^{-\varphi(x,i)},\ t)\ dt}$$

The correspondence of the above formula with the Laplace transform (with s being a positive number) is striking:

$$\boldsymbol{\int^{\infty}_{t=0} e^{-st}\ F(t)\ dt}$$

although the operational rules here need more clarification. For example, the Rule of Addition, the Rule of Scale and the Rule of Shift in the Transform Function and/or the Shift in the Original Function, are interesting tools (14). Here, the Rule of Scale is given as an example, whereby a variable t can be replaced by a scalar product $at = z$ (after 14). This will re-appear in the adaptation to different time scales (see below).

$$\mathcal{L}\{F(at)\} = \int_0^\infty e^{-st}F(at)dt$$

$$\mathcal{L}\{F(at)\} = \frac{1}{a}\int_0^\infty e^{-\frac{s}{a}z}F(z)dz$$

$$\mathcal{L}\{F(at)\} = a^{-1}f(\frac{s}{a})$$

The existence of a Laplace transform, moreover, can be inferred from the rule that, for functions belonging to a class A, such a Laplace transform exists. Namely, when $F(t)$ is of some exponential order as $t \to \infty$, such that $|F(t)| < M^{et}$ for all t $\geq T > 0$ (14). For functions of this class A, the integral is then the sum of two integrals, the first being integrable when $F(t)$ is piecewise continuous, the second vanishes for $s > \sigma$. Allowing $s \to \infty$ the result was found that $lim\ s \to \sigma\ f(s) = 0$, whenever $F(t)\ \mathcal{E}$ class A (14). The latter properties can be attributed to the overall survival prognosis described above ($P_{overall}$). Given a fractal distribution (in space as well as in time) of a species in an ecosystem, the piecewise-continuity property is applicable too.

Obviously, when extrapolating the survival probability from a multicellular organism to a multi-species ecosystem, it is no longer the individual’s survival rate that counts, but the probabilities of individuals of a species to give birth to their offspring before being killed by a predator (most notably from another species), from disease or by some accidental form of death. So, the death of an individual may become a life-saving event for its predator. (The interaction with the human species, in particular, will receive some attention below). Moreover, not only the survival rates of different species (as a collective of individuals following a certain demographic distribution), but also the mixture of different species is at stake (see also **Implications for biodiversity resilience and ecosystem robustness**). Therefore, some biodiversity index as Yule’s 'characteristic' (Y) (19) has to be involved, of the form:

$$Y = N^2 / \Sigma N_i (N_i - 1)$$

The latter expression in fact gives an estimate of the proportion of the number of individuals (squared) found in an area, divided by the sum of squares of the abundance numbers of different species, resulting in an estimate of the diversity of the sample (7). The importance of an area correlate of the survival probability is discussed below. An important caveat should be given to the possible impact of random sampling (within a limited array of trophic levels) giving rise to the so-called 'gate keeping' bias, as explained previously (20). Moreover, biodiversity estimation does not coincide with counting the animals in a zoo, or, it is not optimized by keeping the maximal number of species per km squared (20).

Finally, it is a typical trait of natural ecosystems, obtained from a vast experience through generations of observers, that the stability of an ecosystem is not only based on the presence of specific prey animals or plants, but also on the occurrence of species from a number of different trophic levels. These include e.g. the primary producers, the herbivores, predators, top-predators, and also organisms involved in decay of organic material (reducers). When regarded at a global dynamic scale, this leads to quite unmanageable formula's for the (global) biodiversity decline (or resilience) in time (6).

How to define the biodiversity at a local scale or the local biodiversity, taking advantage of its fractal nature? As beautifully explained by Benoit B. Mandelbrot (1924-2010), some self-evident observations paved the way for the development of a fractal theory of the geometry of nature (21). Or, the fractal nature of the living biosphere following the evolutionary process, mirrors the fractal nature of the Earth's geology (22).

For instance, taking Richardson's so-called coastline function as example, for the total length (L) of the coast of for instance Great Britain, in terms of a number (F) of fragments of length ε is described by:

$$L(\varepsilon) = F\,\varepsilon^{1-D}$$

with D the so-called *fractal dimension*(21). According to Mandelbrot, the following, general expression can be used for a number of N fragments of length r, D being called the *similarity dimension*:

$$\log r(N) = \log 1 / N^{1/D} = -(\log N) / D$$

In order to transfer this characterization of a fractal geometry to a dynamical ecosystem, for instance the fractal biodiversity interface (7), N may represent the number of interacting species (competitors, predators, parasites, etc.) in a characteristic spatial environment with characteristic size D. We will come back to the relation between trophic level, habitat-characteristic size and characteristic time-scale (see below).

How to describe the dynamics of this biodiversity interface in a stressed ecosystem, such as in the present biosphere?

A well-known application of Laplace transformation is the technique of time-iterative convolution of correlation functions (23). This offers an interesting instrument for the dynamic behavior of ecological networks at a local scale. In simplified form, when the local biodiversity is described in terms of functions $f_1(t), f_2(t)$, the convolution transformation is defined as the product of the Laplace transforms (F1, F2), as followed:

$$F_1(p)F_2(p) = \mathcal{L}\left\{\int_0^t f_1(\tau)f_2(t-\tau)d\tau\right\} = \int_0^\infty e^{-pt}\left\{\int_0^t f_1(\tau)f_2(t-\tau)d\tau\right\}dt$$

A convolution of *f1* (*t*) and *f2* (t-τ) is possible, wherein t and τ (tau) may also refer to two different time scales, t being the ecosystem time (or real time). On the other hand, τ (*tau*) represents the perceived time (of an animal) before it is disturbed by human interference (in the sense of leading to the animal's death by hunting, its being captured or simply disturbed by human encounter). Consequently, τ (*A*) depends on the territorial size of the animal's habitat (and by generalization the species habitat) and the distance to the border (interface). As a working hypothesis, we suggest that τ (*A*) is proportional to a characteristic size of the animal (*L*) and its velocity (*L/t*), or:

$$\tau(A) \sim \frac{\alpha L^{\beta}}{\frac{L}{t}} = \alpha L^{\beta-1} t$$

with α, β.., the species-specific parameters (describing how the body size, velocity and area are interrelated for a species), α> 0 and β>1. For plant species, velocity is replaced by the time window (duration of blossoming, size of the seeds, etc.) of the flowering of the species and spreading of the seeds. Likewise, the dependence of one species towards another (that in many cases are required for a species in order to survive), may also be expressed in terms of a series of power terms or constituting a polynomial in N terms (N for number of species concerned).

To account for different habitat sizes and correlated time scales, an iterative procedure is a well-known approach. Hereby, a combined approach, taking advantage of the Laplace transforms of polynomials and of integrals, in theory, can be considered (14) (not worked out in this paper).

Finally, the successive dynamic descriptions of the local biodiversity correspond to the probabilities of transitions in a Markov chain. Here, a strong analogy exists with the iteration function that subsides the Mandelbrot set (M), namely following the iteration (6),(21):

$$P_c: z \rightarrow z^2 + c$$

Resulting in the set M of points of the set of complex numbers C, that fulfills the supremum norm:

$$M = \{ c \,\mathcal{E}\, C: \sup P_c^{\infty}(0) < \infty\}$$

The iteration function P_c tends to a finite maximum (depending on the value of c), that may be useful as a boundary condition in terms of the fractal dimension (D) and number of interacting species (N) (see above). In theory, but only then, this characteristic may enable to extrapolate the local behavior of a fractal network to the generalized network characteristics, such as the critical percolation probability, the critical dimensionality and the biodiversity resilience characteristics of a (local) ecosystem (6) (see also **Implications for biodiversity resilience and ecosystem robustness**).

Implications for biodiversity resilience and ecosystem robustness

The problem of finding a mathematical, in particular a numerical correlate of a typical phenomenon in nature, reminds us of the example of the ratio between the actual length of a river 'from source to mouth' and their direct length 'as the crow flies' (24). In fact, this ratio appears to approximate the value of the number π (= 3,141592…) (24), a finding that in the case of the river ratio was believed to be the result of 'a battle between order and chaos'. According to Simon L. Singh (°1964, Somerset, UK), the physical explanation of the course of the river allegedly was formulated by Albert Einstein (1879-1955) as a combination of a 'tendency towards an ever more loopy path' (designated as 'chaos') and the natural process of curtailing the chaos by short-cutting the loops and the formation of 'ox-bow lakes' (24).

Given the latter metaphor of the length ratio of the winding river (actual length vs. bird's eye-view), the following intuitive drawing of a biodiversity deployment is illustrative (Fig. 1). Between the extremes where an area is occupied by merely one species (bringing its competitor[s] to the edge of extinction) and the opposite situation (where that very species becomes extinguished), an infinite array of intermediate states may be drawn. These two extreme positions are reminiscent of the so-called hysteresis cycle, a typical loop cycle in bi-stable ecosystem dynamics (25). Interestingly, in the case of plant-pollinator systems which are extremely important for global biodiversity and food security, H. (26) found weak empirical evidence for an effect, although statistically non-significant, of the degree of network connectedness. Using synthetic networks generated by random plant-pollinator links (with experimental data obtained from the isles of Mauritius), they were able of further refining and strengthening their analysis. Moreover, they concluded that higher degrees of biodiversity and network connectedness enhance the system's resilience to disturbance (26).

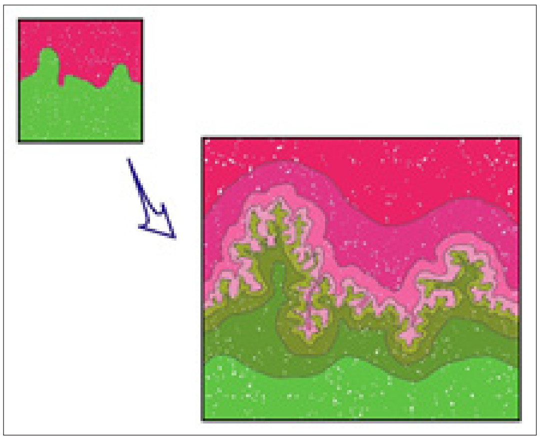

Figure 1: Symbolic representation of the fractal notion of Woodland-Grassland interface (also applicable to other border ecosystems, like Coral Reefs, Mangroves forests). (*see ref. 7 for more ecological details*).

In fact, this property of network connectedness has to be expanded further, as it appears that 'rich' ecosystems may often generate biodiversity hot spots (6). Otherwise, in rather desolate habitats, like in the (sub)arctic and alpine environments, still a fewer number of species may constitute (stable) ecosystems (20).

In contrast to Huang and D' Odorico's mean field approach (25), (26) the fractal approach starts from the dynamics of the border interface (7). In the 2D-representation of the interface (Fig. 1), e.g. between woodland and grassland (or mangrove coastlines, coral reefs), time (t) can be added as a third dimension, or replace one of the spatial dimensions. The undulating borderline may therefore also represent the dynamic alterations of the ecosystem. Taking one step further, the 2-dimensional borderline also translates into the survival pathway (as a whole) of a number of interacting species in their environment (Fig. 2).

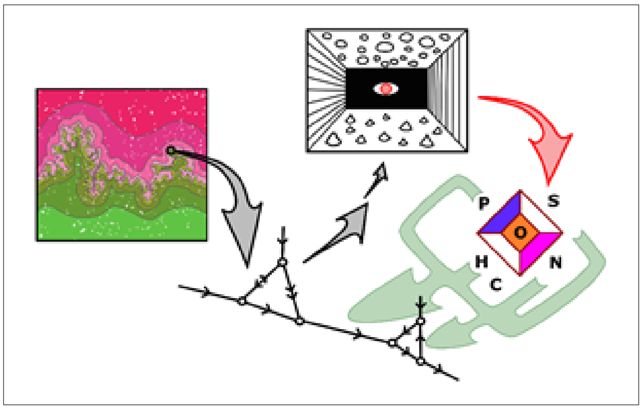

Figure 2: Deconstruction of the fractal Biodiversity interface (see Fig. 1). The interface consists of interacting plant and animal species, their interactions here being represented by so-called open trails after (16). Moreover, interactions can be further analyzed at subsequently smaller scales, until the micro-scale of interacting chemical elements (C,N,O,S,P), the proportion between them playing also a key role in enabling growth and metabolism of the living ecosystem (see 35).

However, the (theoretical) applicability of the methodology of Laplace transformation also recalls an encounter with the Demon of Laplace. Here, it is applied to the macroscopic realm of the ecosystem's biodiversity. Transferred to the atomic level, Laplace's demon signifies that if someone (called the demon) "knows the precise location and momentum of every atom in the universe, their past and future values for any given time are entailed", and can be calculated from the laws of Newtonian mechanics (27). Similar to the question of predictability of the fundamental particles in the universe, in which case quantum mechanics and the laws of thermodynamics have counter-indicated this idea of Laplacian determinateness, also here some level of indeterminacy is perceived.

This indeterminacy recalls some analogy with the entropy notion (see **Entropy: definitions and relation between area and amount of information**). Although we may not have a microscopic or nanoscale correlate of the entropy of a local ecosystem at the border/interface, we may - for the reason of argument - define a change of entropy in terms of the linear displacement of a new element, say an individual of/or a (new) species entering the ecosystem's environment, towards the border/interface. This is similar to the arguments of Bekenstein and Verlinde (see **Entropy: definitions and relation between area and amount of information**). For the positive relation between entropy (increase) and distance to the border area (in Bekenstein's model it is the area of a black hole) (5,13), the linear coefficient needs to be positive too. This would implicate that the entropy of the interface is augmented, when species are added and the total number of degrees of freedom of the interface/border increases (or the number of nodes in the scale-free networks discussed below). A different perspective is that when the exact position of an individual (or species or node belonging to the network) is established at a certain moment at a certain distance separated from the interface. The question then is whether the number of degrees of freedom of the remaining elements of the ecosystem is lowered, or not. For instance, the interference with one individual (or species) may have an effect on the other individuals/species (that may eventually cross the border-interface and are no longer in sight). So, adding or removing an element both may result in enhancing the entropy of the whole, which produces a paradox with the foregoing (when an element is added, the entropy grows). A somewhat naïve intuitive explanation, especially adapted for the rather myopic naturalist-photographer, is that of making photographs of an object that is somewhat separated from the actual focal distance (distance between the photographer and the focal plane of a natural scene). We can see the object, but it is blurred compared to the rest of the image in focus. Or vice versa, the selected object is sharp and the remaining lot is blurred. Defining the actual location of one species or individual, interferes with the biodiversity of the system, but doesn't lower the entropy of the residual group (of species/individuals).

Apart from the criticism that for the second law of thermodynamics to hold, a closed system is needed (and the planet can be regarded as such, but by definition not a local border/interface). The 2 points of view pictured above nevertheless are truly conflicting, if we expand the notion of interface to the entire biosphere of planet Earth. Moreover, an important contribution of migration of animals, plant seeds, etcetera, to biodiversity has been well-established (20). Apparently, this paradox illustrates two conflicting paradigms. On the one hand, the all-knowing, anthropogenic or Laplacian view on biodiversity as the sum of all positions of all individuals of all species (in the global biosphere, or in a distinct ecosystem), versus the global state of a by far still unknown system, designated as Nature.

Can this paradox be avoided at a more abstract level? That is an interesting question, which so far remains unanswered.

Returning to a true ecosystem, we have to realize that survival until the moment of successful reproduction enables to extrapolate from individuals to the species involved (i.e. when also its offspring is ready for reproduction: so early death of the offspring doesn't count as successful!). Of course, we have to make abstraction of more biologically relevant data, in order to make a predictive model work. For instance, when not all male/female individuals of a species take part in reproduction (because of some mechanism of competition like in deer herds, or in the [Black] Grouse [*Tetrao tetrix*] or in many social insects..), the individuals that don't actually have offspring may equally well contribute to the struggle to transmit the best characteristics to the next generation. Therefore, even a predator feeding on the enfeebled or diseased individuals, eliminating a surplus of one species at the cost of the environment, may finally help the survival of the prey species. Finally, if one simplifies the complex ecological interaction pathways between species as either subsistent or noxious to the co-habitation of these species, this would allow for the construction of multi-nodal complete graphs (28). The application into ecology of Combinatorial Mathematics, such as the so-called *Ramsey Numbers* (28), may imply that interactions between two species can be interpreted as either predatory, competitive (for the same food source, shelter or other commodity), commensal or neutral (see **Possible application of Combinatorial Mathematics in Ecology**). However, the reciprocity of these interactions in general is not guaranteed, or it is not consistent for all age groups or environmental conditions (such as with famine or abundant food supply), let alone the numerous reported cases of cannibalism within the own species (e.g. in lions, bears, etcetera). So, mathematical modelling necessarily detracts from the biological complexity of the real ecosystem.

One or a few important result(s), however, that may be derived from complexity analysis, is (are) the notion of introducing (adding) or eliminating (subtracting) small numbers of species from an ecosystem, and the notion of critical dimensionality of the network. In theory, a mean field value of the critical dimensionality (of a hierarchical cluster) (6) may be defined in analogy with the fractal formulas for the critical percolation of a network as described by Mandelbrot (21). In practice, however, these theoretical notions are hard to define in an actual ecosystem with multiple trophic levels, where many species may still be not yet known nor determined, or simply not accounted for in the actual biodiversity estimation (20).

Another question has been raised, derived from certain approaches to 'manage' the species composition of border habitats (e.g. between forest and grassland) by controlling, resp. eradicating the wanted resp. unwanted seedlings, etc. It is related to the important question: what is the effect of adding or removing a species into/from a habitat, in particular the border zone, on the biodiversity of the border zone? As we discussed in this paragraph, there indeed seem two possible approaches to estimate biodiversity, one from a deterministic, Laplacian viewpoint, the other from a more observational, stochastic a *posteriori* approach.

Possible Applications of Combinatorial Mathematics in Ecology

In combinatorics, F.P. Ramsey's (1903-1930) theory (29) has been suggested to offer a wide range of possible applications into probabilistic problem solving (28). In a nutshell, Ramsey's theorem, based upon which later the term *Ramsey's Theory* was coined (30), stated that in any sufficiently large system, how chaotic it may seem, there always have to be certain patterns of order (to be discerned). In graph-theoretic forms, the theory states that in any sufficiently large complete graph (meaning that all nodes – or vertices - of the graph are connected by so-called edges), where edge labeling has been applied (for instance with two or more distinct colors), a least positive integer exists indicating the occurrence of monochromatic subgraphs (e.g. triangles). The extension for any finite number of (more than two) colors states: "for any given number of colors (c), and any given integers ($n_1, n_2, n_3, \dots n_c$), there is a number R ($n_1, n_2, n_3, \dots n_c$), such that if the edges of a complete graph are colored with c different colors, then for some i between 1 and c, it must contain a complete subgraph of order n_i whose edges are all of color i (…)" (30).

Applying this combinatorial paradigm, requires some simplification from the previous complexity of ecological interactions (**Implications for biodiversity resilience and ecosystem robustness**). The most simple approach is to consider only two colors or flavors of interspecies interaction, let us say the 'interdependent' (one species is a predator or feeds on another species, or one species limits the growth of the other) and 'independent' or not-interfering/neutral type of interaction. In a more elaborate version of the application, more types of interaction and subtler ecological mechanisms could be introduced. A direct and simple result is that the minimum number of species in such a system would require a Ramsey number of 6 (for two colors), or $R(3,3) = 6$ for $c = 2$ (Ramsey's Theorem, 2023). It is also noteworthy that it has been proven that $R(4, 4) = 18$ and $R(4, 5) = 25$, whereas the exact value of $R(5, 5)$ is still unknown (but proven to be somewhere between 43 and 48) (30). Translated into an ecological argument, this would mean that at least six species are needed to contain two mutually independent interacting species groups (of at least 2 'colors' of either interacting or non-interacting species). From this interpretation, however, it is hard to infer a conclusion stating that ecosystems of fewer number of species (say five) cannot thrive. For, complete graphs imply that between each pair of species only one type of 'relation' may exist, which is far from the experienced flexibility of Nature.

Although the pioneering work of Paul Erdös (1913-1996) and colleagues (31), already in 1947 led to an important result for introducing the probabilistic method in calculating exponential lower (and upper) bounds for the Ramsey numbers, many improvements of the theory are awaited to date (30). Progress has been made by the introduction of *Induced Ramsey Numbers* (on a graph H) ($r_{ind}(H)$) (30), where also not-complete graphs could be included into the analysis (including cycles, paths or stars on k vertices, where r_{ind} (H) is linear in k) (32). Also the so-called 1-degenerate graphs or trees were included in the Induced Ramsey Number theory, although the Burr-Erdös conjecture for the usual Ramsey numbers gave contrasting results on the linearity of r (H) for trees (or, so-called 'open' graphs), compared to the r_{ind} (H) (30, 31). For the time being, the still unproven Erdös conjecture that $r_{ind}(H) \leq 2^{ck}$ is a good starting point providing an upper bound for general induced Ramsey numbers. For lower bounds, 'at least' the Ramsey numbers can be used, for which in some particular cases the value is known (see above).

In scale-free networks, Albert and Barabási (33) demonstrated that the connectivity distribution follows a power law, when certain restrictions were met for the number of the nodes, corresponding to a low connectivity state typical of an evolving network (23). There is only a faint similarity between the former bounds and the lower, resp. upper bounds in Mandelbrot's interpretation of percolation theory (21). With a critical probability derived from Bernouilli's (1700-1782) notion of percolation (p_{crit}), Mandelbrot defined an upper bound for the critical fractal dimension (D_{crit}) from:

$$\mathbf{b}^{D_{crit}} > b^{E} + \tfrac{1}{2}\, b^{E-1}$$

(with b being the lattice base giving rise to b^E subdivisions or intervals) (6). A lower bound was derived for $b>>1$, namely $D_{crit} > E + \log_b p_{crit}$ (6).

The notion of lower/upper bounds may become useful to find a limit for the iteration procedure by calculating successive Laplace transforms (convolutions over a period long enough to allow for interactions between all species in an ecosystem, as well as spatial dimensions small enough to account for all trophic levels). Taking the analysis back to the realm of living ecosystems, the probabilities of interactions between species (being either dependent, neutral or otherwise) obviously depend on the probabilities of one species encountering another species. For large solitary carnivores (like the Eurasian lynx [*Lynx lynx*], Snow leopard [*Panthera uncia*] (34), ...), where animals sometimes need months to cross their entire territory, where only occasionally individuals of the same species are met (which is essential for reproduction), these occurrences may be extremely rare indeed. Otherwise, large herbivores tend to graze on herbs and grass almost constantly, with only some time for ruminating or resting in between several grazing periods. Therefore, a time-integrated averaging of the occurrence of interspecies interactions is needed, taking into account the abundance, territorial dimensions (and the time needed to cross one's territory), reproductive success, et cetera (e.g. large animals have a longer life span, but procreate less offspring, according to the Janis – Carrano hypothesis) (7) (see also **Time-integrated convolution of fractal functions using Laplace transformation**).

Concluding Remarks

Starting from Schrödinger's metaphor of a cat that could be both death or alive depending on the probabilistic nature of a quantized decay process (see **Introduction**) and Bekenstein's notion of a relationship between entropy and surface area (of a black hole), we proposed the question whether the uncertainty of biodiversity alterations could be linked to an entropy or negentropy definition related to evolving ecosystems.

Transference into network connectivity theory, implicates this question is translated into: what is the effect of adding nodes (vertices) to a network on system's biodiversity ? And what is its effect on the system's entropy? These questions are especially relevant in case of attempts to exert a 'controlled' management of an ecosystem, in particular the ecosystem's interface or boundary between different habitats.

Moreover, these questions reflect two different scenarios of ecosystem dynamics. The classical scenario is that of a natural evolution, meaning that natural selection is a means for animal species to find highly or unusual conditions for survival. This is in fact a race, which in time is won by the (individuals of a) species that are best adapted to these 'unusual' conditions. For species that are filling in a new or empty niche, the probability of finding enough prey/food/shelter before the time window of passing their genes to the following generation, determines their adaptive fitness.

The other scenario is that of the 'anthropogenic' collapse of the biosphere: what is the probability of finding animals in a survival area controlled by human surveillance, in terms of numbers of species and individuals of a species (the relative abundance) in a given area of an ecosystem?

In the second scenario, it is not surprising to find a general impairment of plant and animal biodiversity in a wide range of biotopes, the many exceptions put aside. We previously discussed the anthropogenic nature of the distinctions and categories developed in modern scientific literature, in order to tackle the multiple levels of 'biodiversity' (35). The question whether adding or removing a species (as a network vertex) augments the entropy/negentropy of the biosphere, not only has a methodological and theoretical connotation, it is also of practical importance for the biosphere's sustainability. The question how the entropy of such an ecosystem change is related to the Second Law of Thermodynamics, however, is far from evident. In the present review, we have abstained from commentaries on previous argumentations on temporal windows for entropy decreases, for instance presented by Ilya Prigogine (1917-2003) (36). Many reasons can be given for this abstinence, such as the absence of a causality horizon, which element plays an important role in Bekenstein's theory of black holes.

In the case of the biodiversity of the interface between different ecosystems, there is no absolute barrier between either regions separated, so there is in fact no preferred time direction of the system dynamics, except for the distinction between the man-made and natural scenarios of system change. So there is also no causality horizon, in the original sense of the notion (4, 13). However, a virtual 'causality horizon' may be discovered here, that has to be regarded as a horizon of a conceptual kind, just as suggested by Verlinde (4, 13) (see **Entropy: definitions and relation between area and amount of information**). Namely, it is rather a 'horizon' between the known or controlled, anthropogenic information (similar to AI), and the unknown spaces of Nature. Or, in the words of Michel Foucault (1926-1984) in *The Order of Things* (37), as quoted before (35): "A dark space appears which is (must be?) made progressively clearer. That space is where 'Nature' resides, (…)" (37).

References

1. Schrödinger, E. (1967). *What is Life? (with 'Mind and Matter' and 'Autobiographical Sketches').* 1967 ed. Cambridge University Press, Cambridge UK (1944).
2. Schrödinger, E. (1935). Die gegenwärtige Situation in der Quantenmechanik. *Naturwissenschaften*, 23, 844-849. https://link.springer.com/article/10.1007/BF01491987
3. Bekenstein, J. D. (1973). Black holes and Entropy. *Physical Review D ,7*(8), 2333-2346.
4. Allaerts, W., & Vellekoop, A. (2012). Gravity demystified: Entropic force and the Emergence of Laws of Newton (Interview with Erik Verlinde). *bi-logical, 4*(1), 9-13.
5. Bekenstein, J. D. (2003). Black holes and information theory. *arXiv: quant-ph/0311049v1*, https://arxiv.org/abs/quant-ph/0311049
6. Allaerts, W. (2020). Estimating Biodiversity and the Fractal Nature of Ecosystems. *International Journal of Bioinformatics and Computational Biology, 5*(1), 15-24.
7. Allaerts, W. (2023). On Land Use and Biodiversity Resilience: Hydrology, Temperature Regulation and the Fractal Nature of the Woodland-Grassland Interface. *Austin Environmental Sciences, 8*(2), 1097.
8. Boltzmann, L. (1896). Lectures on Gas Theory (translated by S. G. Bruch). *Dover Publications Inc.*, New York
9. Boltzmann, L. (2000). Entropie und Wahrscheinlichkeit: (1872-1905) (Ostwalds Klassiker der exakten Wissenschaften, ISSN 0232-3419 (German Edition). Deutsch, Frankfurt am Main (ISSN: 0232-3419).
10. Brillouin, L. (1953). Negentropy Principle of Information. *Journal of Applied Physics, 24*(9), 1152 – 1163. DOI : https://doi.org/10.1063/1.1721463

11. Cercignani, C. (2003). Boltzmann – Un genio nell' Austria felix. Le Scienze, Roma (Dutch transl. E. Maris [2013]. Boltzmann – Briljant maar onbegrepen natuurkundige, p. 49-57. Veen Media, Amsterdam)
12. Susskind, L. (2008). The Black Hole War. My battle with Stephen Hawking to make the World Safer for Quantum Mechanics. *Little, Brown & Co*, New York, London
13. Verlinde, E. (2010). On the Origin of Gravity and the Laws of Newton. *arXiv*, DOI: https://arxiv.org/abs/1001.0785v1
14. Williams, J. (1973). Laplace transforms. Series: Problem Solvers. George Allen & Unwin Ltd., London.
15. Allaerts, W., Wouters, A., van der Massen, D., Persoons, A., & Denef, C. (1988). A diffusion-adsorption model for the computation of the amount of hormone around a secreting cell, detected by the Reverse Hemolytic Plaque *Assay. J. theor. Biol, 131*(4), 441-459. DOI: https://doi.org/10.1016/s0022-5193(88)80040-7
16. Ma'ayan, A., Jenkins, S. L., Neves, S., Hasseldine, A., Grace, E., Dubin-Thaler, B., Eungdamrong, N.J., Weng, G., Ram, P.T., Rice, J.J., Kershenbaum, A., Stolovitzky, G.A., Blitzer, R.D., & Iyengar, R. (2005). Formation of regulatory patterns during signal propagation in a mammalian cellular network. *Science, 309*(5737), 1078-1083. DOI: https://doi.org/10.1126/science.1108876
17. Shu, J., Wu, C., Wu, Y., Li, Z., Shao, S., Zhao, W., Tang, X., Yang, H., Shen, L., Zuo, X., Yang, W., Shi, Y., Chi, X. Zhang, H., Gao, G., Shu, Y., Yuan, K., He, W., Tang, C., Zhao, Y., & Deng, H. (2013). Induction of Pluripotency in Mouse Somatic Cells with Lineage Specifiers. *Cell, 153*(5), 963-975. DOI: https://doi.org/10.1016/j.cell.2013.05.001
18. Allaerts, W. (2015). Studying Multiscale Probabilistic and Deterministic Models for the Annotation of Cellular Networks and Tumor Metastases. *Journal of US-China Medical Science*, 15, 1-11.
19. Williams, C. B. (1946). Yule's 'characteristic' and the 'index of diversity'. *Nature, 157*(482), (1946).
20. Allaerts, W. (2023). The Unified Neutral Theory of Biodiversity and Biogeography Revisited. *Advances in Earth and Environmental Science, 4*(2), 1-10.
21. Mandelbrot, B. B. (1983). The Fractal Geometry of Nature. W.H. Freeman and Company, *San Francisco*.
22. National Geographic. (2008). Snowball Earth (Documentary). IMDb films https://www.imdb.com/title/tt1334816/
23. Allaerts, W. (2019). Estimating the connectivity, continuity and distance norm in mathematical models for community ecology, epidemiology and multicellular pathway prediction. *J. Phys. Conf. Ser,* 1391, 012119.
24. Singh, S. (1997). Fermat's Last Theorem. 2011 ed. Harper Perennial, London, New York, Toronto, Sydney
25. Huang, H., & D'Odorico, P. (2020). Critical transitions in plant-pollinator systems induced by positive inbreeding-reward-pollinator feedbacks. *iScience 23*(2), 100819. DOI: https://doi.org/10.1016/j.isci.2020.100819
26. Huang, H., Tu, C., & D'Odorico, P. (2021). Ecosystem complexity enhances the resilience of plant-pollinator systems. *One Earth, 4*(17), 1286-1296 DOI: https://doi.org/10.1016/j.oneear.2021.08.008
27. Laplace, P. S. (1902). A philosophical Essay on Probabilities. J. Wiley / Chapman & Hall, New York, London (1814).
28. Do, N. (2006). Party problems and Ramsey theory. *Austr. Math. Soc. Gazette, 33*(5), 306-312.
29. Ramsey, F.P. (1930). On a problem of formal logic. Proceedings of the London Mathematical Society 30, 264-286.
30. Ramsey's Theorem (2023). Homepage, http://www.wikipedia.com, last accessed 2023/10/29 https://en.wikipedia.org/wiki/Ramsey%27s_theorem
31. Erdös, P. (1990). Problems and Results on Graphs and Hypergraphs: Similarities and Differences. In: Nešetril, J., Rödl, V. (eds.). Mathematics of Ramsey Theory. Algorithms and Combinatorics (series), pp. 12-28. *Springer*, Berlin, Heidelberg
32. Fox J., & Sudakov, B. (2008). Induced Ramsey-type theorems. *Advances in Mathematics, 219*(6), 1771-1800. DOI: https://doi.org/10.1016/j.aim.2008.07.009
33. Albert, R., & Barabási, A.-L. (2000). Topology of Evolving Networks: Local Events and Universality. *Physical Review Letters*, 85, 5234-5237. https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.85.5234
34. Amiguet, M., Munier, V., & Tesson, S. (2021). The Velvet Queen (La Panthère des neiges). Motion picture distributed by Oscilloscope Laboratories, Cannes, France
35. Allaerts, W. (2021). Deconstructing Self-Similarity and the Four Dis-similitudes of Biodiversity. *Philosophy International Journal, 4*(2), 16000180.
36. Prigogine, I., & Stengers, I. (1984). Order out of Chaos. (Dutch Transl. M. Franssen and M. Morreau [1985], Orde uit Chaos. De nieuwe dialoog tussen de mens en de natuur, pp. 298-299. Bert Bakker, Amsterdam)
37. Foucault, M. (1970). The Order of Things. An Archaeology of the Human Sciences. Vintage Books, Random House Inc., New York.