Research Article

ISSN 2767-3901

International Journal of Theoretical & Computational Physics

Interior Solution of Einstein's Field EquationFor a Homogeneous Spherical Massive Bodies Whose Tensor Field Varies in Radial Size and Time Using Riemannian Golden Metric Tensor

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Submitted : 19 Jun 2025 ; Published : 19 Sept 2025

Citation: Ode, A. I. et al., (2025). Interior Solution of Einstein's Field Equation For a Homogeneous Spherical Massive Bodies Whose Tensor Field Varies in Radial Size and Time Using Riemannian Golden Metric Tensor. *I J T C Physics, 6*(3):1-3.

DOI : https://doi.org/10.47485/2767-3901.1066

Abstract

The modification of Newton dynamical theory of gravitation (NDTG) by Einstein's geometrical gravitational field equations (EGGFE) do not invalidate Newton dynamical theory of gravitation or require its replacement instead the Einstein's field equations differs meaningfully from those of the Newton's field equations only for object moving at relativistic speed. In this article, an exact analytical solution of Einstein's geometrical gravitational field equation interior to a static homogeneous spherical bodies whose tensor field varies with time and radial distance was constructed and solved. It was observed that within the interior field, the solution converges to Newton dynamical scalar potential which is thus an extremely discovery with the reliance on two arbitrary function. The result obtained in the limit of weak field is equal to laplacian equation which does not differ significantly from Newton dynamical theory of gravitation. The solution further confirms the assumption that Newton dynamical theory of gravitation is a limiting case of Einstein's geometrical gravitational theory of gravitation.

Keywords : Newton's theory, Einstein's theory, Ricci tensor, energy-momentum tensor, general theory of relativity.

Introduction

General Relativity, also known as the general theory of relativity and Einstein 's theory of gravity, is the geometric theory of gravitation published by Albert Einstein in 1915 and the current description of the gravitation in modern Physics (Bergmann, 1947). General relativity generalizes special relativity and refines Newton law of universal gravitation, providing a unified description of gravity as a geometric property of space and time or four dimensional space-time (Weinberg, 1972).

The term General Relativity is the most widely accepted theory of gravitation (Howusu, 2010; Chifu, 2012). The equations are in the form of tensor equation which related the local space-time curvature expressed by the Einstein tensor with the local energy and momentum within that space-time expressed by the stress-energy tensor (Misner et al., 1973).

After Einstein's publication of geometrical gravitational field equation in 1915, the search for their exact and analytical solution began for all the gravitational field in nature (Howusu, 2010; Chifu & Howusu, 2009; Chifu, 2012; Maisalatee et al., 2020). The first to construct the exact solution of this field equation in a static and pure spherical polar coordinates in 1916 was Schwarzschild by considering bodies which are astrophysical such as the sun and stars. In Schwarzchild's metric, the tensor field differs with radial distance only.

A new method and approach was introduced to formulate exact analytical solutions (Chifu & Howusu, 2009) as an extension of Schwarzschild's method. This new approach took into consideration the fact that tensor field of astrophysical bodies does not depend on radial distance only as indicated in Schwarzschild's equation. In this article, we show how exact analytical solution of the interior field equation can be constructed in the limit of c0 in a gravitational field for time varying spherical massive bodies using the new method and approach.

Formulation of Einstein equation

The Einstein's field equation (EFE) interior to a homogeneous spherical distribution of mass is given by (Misner et al., 1973; Tajmar, 2001; Howusu, 2008; Chifu & Howusu, 2009).

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{4\pi G T_{\mu\nu}}{c^4} \qquad (1)$$

Where $T_{\mu\nu}$ is the energy-momentum tensor due to any distribution of mass or pressure
G is the universal gravitational constant.

Consider a homogeneous mass distribution in a weak field limit. We can neglect the contribution from the source, the energy-momentum tensor given by

$$T_{\mu\nu} = \frac{1}{2} \rho_0 c^2 \qquad (2)$$

Where,

p_o is the density

c is the speed of light in vacuum.

Now substituting equation (2) into (1) yields equation (3)

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{4\pi G \rho_0}{c^2} \qquad (3)$$

It is observed (Misner et al., 1973) that the exterior field equations along the G_{11}, G_{22} and G_{33} converge within the exterior field, similarly along the interior field.

Substituting equation (5) and (6) into (4) equation gives

$$G_{00} = -\frac{2}{c^2}\left[1+\frac{2f(t,r)}{c^2}\right]\frac{\partial^2 f(t,r)}{\partial r^2} - \frac{4}{c^2 r}\left[1+\frac{2f(t,r)}{c^2}\right]\frac{\partial f(t,r)}{\partial r} + \frac{2}{c^4}\left(\frac{\partial f(t,r)}{\partial r}\right)^2 + \frac{1}{c^4}\left[1+\frac{2f(t,r)}{c^2}\right]^{-1}$$
$$\left(\frac{\partial f(t,r)}{\partial r}\right)^2 \frac{2}{c^4}\left(\frac{\partial f(t,r)}{\partial t}\right)^2 - \frac{3}{c^4}\left[1+\frac{2f(t,r)}{c^2}\right]^{-2}\left(\frac{\partial f(t,r)}{\partial t}\right)^2 + \frac{1}{r^2}\left[1+\frac{2f(t,r)}{c^2}\right]^2 = \frac{4\pi G \rho_0}{c^2} \qquad (7)$$

Introducing a Laplacian operator yields

$$\nabla^2 f(t,r) - \frac{1}{c^2}\left[1+\frac{2f(t,r)}{c^2}\right]^{-1}\left(\frac{\partial f(t,r)}{\partial r}\right)^2 - \frac{1}{2c^2}\left[1+\frac{2f(t,r)}{c^2}\right]^{-2}\left(\frac{\partial f(t,r)}{\partial r}\right)^2$$
$$-\frac{1}{c^2}\left[1+\frac{2f(t,r)}{c^2}\right]^{-1}\left(\frac{\partial f(t,r)}{\partial t}\right)^2 + \frac{3}{2c^2}\left[1+\frac{2f(t,r)}{c^2}\right]^{-3}\left(\frac{\partial f(t,r)}{\partial t}\right)^2 - \frac{c^2}{2r^2}\left[1+\frac{2f(t,r)}{c^2}\right] = \frac{4\pi G \rho_0}{c^2} \qquad (8)$$

In the weak field limit of the order c0, equation (8) becomes

$$\nabla^2 f(t,r) = \frac{4\pi G \rho_0}{c^2} \qquad (9)$$

Equation (9) is in line with the concept of general relativity, which is equals to Laplacian equation and has a gravitational scalar potential of two functions.

Conclusion

From the result obtained in equation 9, we have established the fact that for a weak gravitational field, the result of Einstein's geometrical gravitational field equations does not differ significantly from Newton dynamical theory of gravitation. But for intense gravitational field, the result diverges from that of Newton's gravitational theory because of additional correctional terms which are not found in the existing once, thus equation 9, is the Newton dynamical scalar field equation. It is indeed a profound discovery, it confirms the assumption made by (Sarki et al., 2018); that Newton dynamical theory of gravitation (NDTG) is a limiting case of Einstein's geometrical gravitational field equations (EGGFE), and this gives more light on the report of (Kumar et al., 2012). It Experimentally shows equivalence principle of physics with the dependency of the gravitational scalar function on time and radial distance only.

For mathematical convenience, we choose G_{00}

Hence the field equation is given by

$$G_{00} = R_{00} - \frac{1}{2} R g_{00} = \frac{4\pi G \rho_0}{c^2} \qquad (4)$$

The coefficient of affine connections of this field constructed were used to construct the Ricci tensor and the curvature scalar given respectively as:

$$R_{00} = \frac{12}{c^4}\left[1+\frac{2f(t,r)}{c^2}\right]^{-2}\left(\frac{\partial f(t,r)}{\partial t}\right)^2 - \frac{3}{c^2}\left[1+\frac{2f(t,r)}{c^2}\right]^{-1}\frac{\partial^2 f(t,r)}{\partial t^2}$$
$$-\frac{1}{c^2}\left[1+\frac{2f(t,r)}{c^2}\right]\frac{\partial^2 f(t,r)}{\partial r^2} - \frac{2}{c^2 r}\left[1+\frac{2f(t,r)}{c^2}\right]\frac{\partial f(t,r)}{\partial r} + \frac{2}{c^4}\left(\frac{\partial f(t,r)}{\partial r}\right)^2 \qquad (5)$$

$$R = -\frac{30}{c^4}\left[1+\frac{2f(t,r)}{c^2}\right]^{-3}\left(\frac{\partial f(t,r)}{\partial t}\right)^2 + \frac{6}{c^2}\left[1+\frac{2f(t,r)}{c^2}\right]^{-2}\frac{\partial^2 f(t,r)}{\partial t^2} + \frac{4}{c^4}\left[1+\frac{2f(t,r)}{c^2}\right]^{-1}\left(\frac{\partial f(t,r)}{\partial t}\right)^2$$
$$-\frac{2}{c^2}\frac{\partial^2 f(t,r)}{\partial r^2} - \frac{4}{c^2 r}\frac{\partial f(t,r)}{\partial r} + \frac{2}{c^4}\left[1+\frac{2f(t,r)}{c^2}\right]^{-2}\left(\frac{\partial f(t,r)}{\partial r}\right)^2 + \frac{2}{r^2}\left[1+\frac{2f(t,r)}{c^2}\right] \qquad (6)$$

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