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Coupled Oscillations of Complex Linear-Ring Structures of Dielectric Resonators

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Abstract

Background: *Today, predominantly one- and two-dimensional lattices of dielectric resonators are used and studied. Three-dimensional complex structures have not been studied sufficiently. Meanwhile, the use of three-dimensional lattices provides new possibilities for the design of antennas, multiplexers, sensors and other elements of microwave and optical systems. The complex spatial structure of the DRs significantly complicates their calculations, especially in the cases of lattices built on a large number of elements. In some cases, an effective solution to this problem can be the use of analytical methods for solving Maxwell's equations. One of such cases is the structure consisting linear and ring lattices of dielectric resonators of different classes.*

Objective: *The aim of this study is to obtain general systems of equations as well as their analytical solutions that simultaneously describe the natural oscillations of complex structures containing ring and one-dimensional lattices, as well as obtaining analytical expressions for the amplitudes and frequencies of the coupled oscillations of different form dielectric resonators, suitable for calculations. To find analytical expressions, perturbation theory and matrix theory are used to construct general solutions of complex systems of linear equations. The end result is the derivation of new general systems of equations, convenient for analysis and obtaining analytical formulas for the frequencies and amplitudes of the resonators.*

Key Findings

1. *The theory of three-dimensional, ring-shaped and one-dimensional complex lattices of complex shapes was first developed.*
2. *The underlying physical laws of interaction between coaxial ring and one-dimensional lattices have been considered.*
3. *New analytical expressions have been extracted for complex frequencies as well as the amplitudes distributions of resonators of complex nested lattices.*
4. *It is shown that the found results in specific cases coincide exactly with analytical solutions of more simple types problems, as well as with the results of numerical calculations.*

Conclusions & Recommendations: *Proposed theory is the basis for building new mathematical models of various types of a broad class of the optical, infrared and microwave devices such as antennas, spatial multiplexers, sensors as well as many other, which are built on the basis of the use of a large number of different dielectric resonators. The developed theory allows also building new more effective mathematical models of various optical communication devices, which are difficult to obtain by numerical methods.*

We Recommend

1. *Use the derived equations in the design of antennas based on a large number of dielectric resonators.*
2. *Use the obtained relationships in the development of spatial multiplexers and access points of the optical communication systems based on dielectric resonators.*

Keywords: Dielectric Resonators; Microresonators; Lattices; Eigen Oscillations; Linear Equation Systems; Eigenvalues; Eigenfunctions; Complex Linear-Ring Lattices.

Introduction

One of the new directions in the evolution of metamaterials implemented on the basis of dielectric resonators (DRs) is the parallel complication of the shape and structure of lattice elements (Tuz & Evlyukhin ,2026) (Jacobsen & Arslanagic, 2025) (Wu et al., 2024) (He et al., 2022) (Kupriianov et al., 2020). (Trubin, 2016; Trubin, 2025; Trubin, 2025; Trubin, 2024). Such complication makes it possible to implement new, more advanced types of filters (Saha et al., 2024) (Lee et al., 2023) (Widaa & Höft, 2022) (Liu et al., 2019). (Zhu et al., 2017). antennas (Kupriianov et al., 2020) (Fesenko et al., 2019). (Luk et al., 2017) (Gangwar et al., 2017) (Prajapati et al., 2014). and many other elements: metalenses (Andreoli et al., 2025) modulators (Prokhorov et al., 2025) lasers (Bidney et al., 2025) (Su et al., 2023) multiplexers (Dai & Bowers, 2014), sensors (Passaro et al.,2012).

The use of different shapes lattices opens up new possibilities for the implementation of antennas, spatial multiplexers, sensors, access points and others. Calculation and analysis of the parameters of such devices are typically performed using numerical methods and often fail to predict their electromagnetic properties. However, in some cases, general solutions for the natural oscillations of lattices with complex spatial shapes, as well as those containing various types of dielectric resonators, can be approximated analytically with high accuracy. One of the types of such complex structures include one-dimensional linear and ring resonator lattices.

The aim of this work is to study the general properties of natural oscillations of combined lattices of dielectric resonators consisting of different linear and ring spatial structures.

This work is based on previously conducted studies of the natural oscillations of complex ring lattices (Trubin, 2025). Expressions for the amplitudes and frequencies of the coupled oscillations were obtained using new analytical solutions, which made it possible to identify and formulate general patterns of their behavior.

Eigen oscillations of the coupled DRs

For calculation and analysis of the amplitudes b_a and frequencies $\omega = 2\pi f$ of the coupled DRs, we used a system of linear equations Trubin (2024):

$$\sum_{s=1}^{N\times J} \kappa_{st}\, b_s - \lambda b_t = 0\,;\ (s,t = 1,\ 2,\ldots,\ N\times J). \qquad (1)$$

Where K_{st} - are coupling coefficients of a s –th and t –th DR modes (Trubin, 2024); for $s = t$, $K_{tt} = i\tilde{k}_t$; where $\tilde{k}_t$ determines by the magnitude of radiation of the t -th DR.

Eigen values of the matrix of coupling coefficients:

$$K = \begin{pmatrix} i\tilde{k}_1 & \kappa_{2,1} & \kappa_{3,1} & \cdots & \kappa_{N\times J,1} \\ \kappa_{1,2} & i\tilde{k}_2 & \kappa_{3,2} & \cdots & \kappa_{N\times J,2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \kappa_{1,N\times J-1} & \kappa_{2,N\times J-1} & \kappa_{3,N\times J-1} & \cdots & \kappa_{N\times J,N\times J-1} \\ \kappa_{1,N\times J} & \kappa_{2,N\times J} & \kappa_{3,N\times J} & \cdots & i\tilde{k}_{N\times J} \end{pmatrix} \qquad (2)$$

Determine complex frequencies ω of the coupled oscillations of the DR system;

$$\lambda = 2(\omega - \omega_0)/\omega_0 \qquad (3)$$

ω_o- real part of the frequency of eigen oscillation of isolated DRs.

Eigen oscillations of a simple ring lattice with dielectric resonator in the center

First we considered a simple case, when a one resonator appears on the axis of a ring lattice, differing in size or in the value of the dielectric permittivity (Fig. 1, a). We assumed that all frequencies of natural oscillations of isolated resonators coincide and are equal ω_0.

We also assumed that the natural oscillations of the resonators are such, that the mutual coefficients between the central resonator (Fig. 1, a) and all resonators of the ring sublattice are the same. We denoted by K_{01} the coefficient of mutual coupling of the central resonator with the resonator of the annular sublattice; and by K_{10} the coefficient of coupling of the resonator of the ring sublattice with the central resonator. For the different DRs: $\kappa_{10} \neq \kappa_{01}$. We proposed that all resonators of the ring sublattice be identical, i.e. their coupling coefficients have the "translation" property: $\kappa_{sn} = \kappa_{|s-n|}$ (s,n = 1,2,...N). We also denoted the coupling coefficient with the external structure of the central resonator by $\tilde{k}_0$, and the coupling coefficient of each resonator of the ring sublattice by $\tilde{k}_t$.

We rewrited equations (1) for the t -th DR and the t-th resonator of the ring sublattice (t = 1,2,,...N). In this case, the equations (1) takes the form:

$$\begin{cases} (i\tilde{k}_0 - \lambda)b_0 + \sum_{n=1}^{N} \kappa_{10} b_n = 0 \\ (i\tilde{k}_1 - \lambda)b_t + \kappa_{01} b_0 + \sum_{n=1(n\neq t)}^{N} \kappa_n b_n = 0 \end{cases} \qquad (4)$$

(t = 1,2,,...N)

Solving a system of equations (4), using the matrix method is cumbersome and does not lead to convenient analytical expressions, so we constructed the required solution using physical assumptions.

- It is not difficult to verify that system (4) has obvious solutions:

$$b_0 = 0\ ;\ b_n^j = b_\infty^{\ j}\ ;\ \lambda = \lambda_o^j \qquad (5)$$

From (4) it follows that these solutions are valid if

$$\sum_{s=1}^{N} b_{os}^j = 0 \qquad (6)$$

where b_{os}^j

$$\mathbf{b}_o^j = \begin{pmatrix} b_{o1}^j \\ b_{o2}^j \\ \vdots \\ b_{oN}^j \end{pmatrix} \qquad (7)$$

$$b_{os}^j = \frac{1}{\sqrt{N}} (\eta_j)^{s-1} \qquad (8)$$

the amplitude of the s-th DR of the isolated known ring sublattice, for which λ_o^j

$$\lambda_o^j = i\tilde{k}_0 + \sum_{s=0}^{N-1} \kappa_s (\eta_j)^s \qquad (9)$$

(j=0,1,..N-1)

frequency parameter of a ring sublattice (Trubin, 2025) for j-th tipe of coupled oscillations $\eta_j = e^{2\pi i \frac{j}{N}}$

As follows from (8), condition (6) is satisfied if $j \neq 0$.

We may also found two more missing particular solutions for azimuthally homogeneous amplitude distributions, assuming $b_n = b_1$ for n = 1, ...N. Then the equation system (4) will be transformed to the form:

$$\begin{cases} (i\tilde{k}_0 - \lambda)b_0 + N\kappa_{10}b_1 = 0 \\ \kappa_{01}b_0 + (i\tilde{k}_1 + \sum_{n=1(n\neq t)}^{N} \kappa_n - \lambda)b_t = 0 \end{cases} \qquad (10)$$

From the condition of existence of a non-trivial solution we obtained:

$$\lambda^{\pm} = \frac{1}{2}(i\tilde{k}_0 + i\tilde{k}_1 + \sum_{n=1}^{N-1} \kappa_n \pm d) \qquad (11)$$

where

$$d = \{(i\tilde{k}_0 + i\tilde{k}_1 + \sum_{n=1}^{N-1} \kappa_n)^2 - 4[i\tilde{k}_0(i\tilde{k}_1 + \sum_{n=1}^{N-1} \kappa_n) - N\kappa_{01}\kappa_{10}\}^{1/2}$$

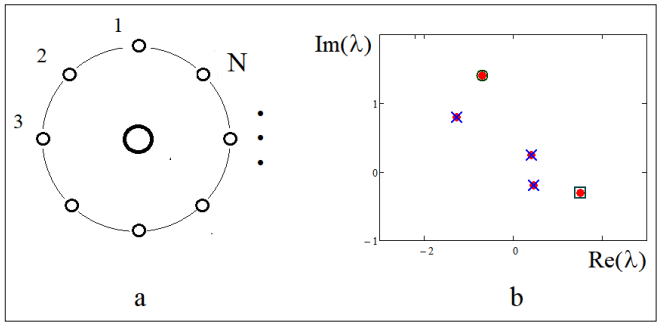
and

$$b_1 = \frac{\lambda^{\pm} - i\tilde{k}_0}{N\kappa_{10}} b_0 \qquad (12)$$

Thus, solutions (5) - (12) determine all N + 1 coupled oscillations of a ring lattice of identical DRs, on the axis of which another resonator with different parameters is located.

It is not difficult to see that if the coupling coefficients between the central resonator and the resonators of the ring lattice vanish, the resonator frequencies and amplitudes obtained from equations (10) and (11) coincide with the frequencies of the isolated central resonator and the frequencies and amplitudes of the coupled oscillations of the ring lattice resonators.

Figure 1: Parameters of the simple Coaxial-ring structure of different DRs



Caption: Simple coaxial-ring structure of different coupled DRs (a). Eigen oscillation parameters of coaxial-ring lattice of different DRs (b): N =7. Here the coupling coefficients are arbitrary complex numbers: k_0 = 0,8; k_0 = 0,3; K_{01} = 0,75 - 0, 5i; K_{10} = 0,2 - 0, 2i ; K_1 = (0,5-2i -0, 25+0,5i 0,15 + 0,1i)^T. In (b) the eigenvalues of the azimuthally non-uniform amplitude distributions are crosses; the eigenvalues of the azimuthally uniform amplitude distributions of the lattice (a) are the circle and small square (b). The results of the numerical calculation are indicated by red dots.

Fig. 1, b shows an example of calculating the eigenvalues using formulas (9) (crosses), (11) (circle and small square) for a ring sublattice of 7 resonators and one resonator on their axis. Eigenvalues calculated numerically are shown by dots. The calculation data demonstrate good agreement between the eigenvalues obtained via the analytical method and the results of numerical calculations of the eigenvalues of the coupling matrix (2).

Thus, we have determined the key physical characteristics of the eigenmodes of coaxial lattices of coupled dielectric resonators. When the mutual coupling coefficients between the central resonator and the resonators of the ring sublattice are equal, the oscillations of the resonator system characterized by an azimuthally asymmetric amplitude distribution are independent of the central resonator's parameters. The resonators of the ring sublattice oscillate independently; their frequencies and amplitudes exactly match those of the resonators in an isolated ring structure. In the case of oscillations with an azimuthally uniform amplitude distribution, the resonators of the ring structure oscillate in phase, and the central resonator participates in these azimuthally uniform oscillations. There are two such azimuthally uniform eigenmodes, and their frequencies and amplitudes are determined by equations (11) and (12).

We will demonstrate that the identified general patterns of coupled oscillations remain valid for more complex linear-ring structures of dielectric resonators.

Eigen oscillations of different Ring-Coaxial Lattices of Dielectric Resonators

We considered a system consisting of several (**M**) ring sublattices with the same number (**N**) of resonators, located coaxially in one plane and with one resonator in the center (Fig. 2, a).

We also assumed that the coupling coefficients between the central 0-th resonator and each of the **M** ring lattices are the same: we denoted the coupling coefficient of the 0-th DR with the resonator of the s-th ring lattice by , and the coupling coefficient of the resonator of the -th ring sublattice with the central, -th DR, by κ_{s0} .

We rewrited equations (1) for the 0-th DR and the *t*-th resonator of the *w*-th ring sublattice:

$$\begin{cases} (i\tilde{k}_0 - \lambda)b_0 + \sum_{s=1}^{M}\sum_{n=1}^{N} \kappa_{s0}\, b_n^s = 0 \\ (i\tilde{k}_w - \lambda)b_t^w + \kappa_{0w}b_0 + \\ + \sum_{n=1,(n\neq t)}^{N} \kappa_{nt}^w\, b_n^w + \sum_{s=1,(s\neq w)}^{M-1}\sum_{n=1}^{N} \kappa_{nt}^{sw}\, b_n^s = 0 \end{cases} \qquad (13)$$

The solutions of system (13), as for one ring sublattice, are also divided into oscillations of two types: “azimuthally uniform”, oscillations with the same amplitudes of resonators in each

ring sublattice, and oscillations with different amplitudes of resonators in each ring sublattice.

Oscillations with unequal amplitudes, taking into account (6), from the first equation (13) $b_o = 0$.

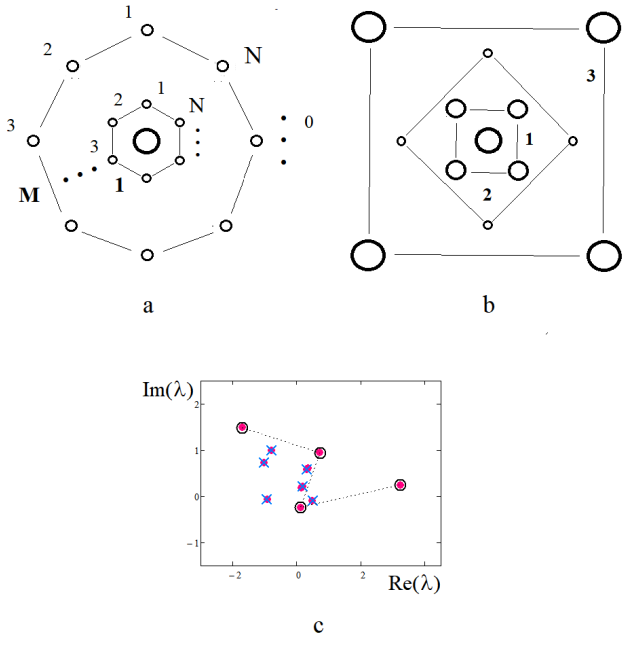
The amplitudes of the remaining resonators are also determined from (13), which is reduced to a system of equations of ring sublattice (Trubin, 2025), by calculating the eigenvalues of the coupling matrix:

$$K^{(J)} = \begin{bmatrix} \lambda_o^{1j} & \sqrt{N}(\mathbf{K}_{21}, \mathbf{b}_o^j) & \dots & \sqrt{N}(\mathbf{K}_{M1}, \mathbf{b}_o^j) \\ \sqrt{N}(\mathbf{K}_{12}, \mathbf{b}_o^j) & \lambda_o^{2j} & \dots & \sqrt{N}(\mathbf{K}_{M2}, \mathbf{b}_o^j) \\ \dots & \dots & \dots & \dots \\ \sqrt{N}(\mathbf{K}_{1M}, \mathbf{b}_o^j) & \sqrt{N}(\mathbf{K}_{2M}, \mathbf{b}_o^j) & \dots & \lambda_o^{Mj} \end{bmatrix} \quad (14)$$

The "vector" of the coefficients of mutual coupling between the u-th and w-th sublattices, represented taking into account the rotational symmetry of the ring sublattice resonators ($\kappa_{ts}^{uw} = \kappa_{|t-s|}^{uw}$):

$$\mathbf{K}_{uw} = \begin{pmatrix} \kappa_0^{uw} \\ \kappa_1^{uw} \\ \vdots \\ \kappa_{N-1}^{uw} \end{pmatrix} \quad (15)$$

Figure 2: Coaxial-multi ring structures of coupled DRs



Caption: Generalized Coaxial-multi ring structure of coupled DRs (a). Eigen oscillation parameters of the Three ring-coaxial lattice of different DRs (b): N = 4; M = 3. The coupling coefficients are arbitrary complex numbers: $\tilde{k}_0 = 0,2$; $\tilde{k}_1 = 0,3$; $\tilde{k}_2 = 0,5$; $\tilde{k}_3 = 0,75$; $\kappa_{01} = 0,6 + 0,35i$; $\kappa_{02} = 0,3 - 0,2i$; $\kappa_{03} = 0,2 - 0,1i$; $\kappa_{10} = 0,5 + 0,3i$ $\kappa_{20} = 0,2 - 0,2i$ $\kappa_{30} = 0,1 - 0,1i$

$$\kappa_2 = \begin{pmatrix} 0,5 - 0,2i \\ 0,2 + 0,1i \\ 0,5 - 0,2i \end{pmatrix} \quad \kappa_3 = \begin{pmatrix} 0,7 + 0,3i \\ 0,5 - 0,2i \\ 0,7 + 0,3i \end{pmatrix}$$

$$\kappa_{12} = \begin{pmatrix} -0,15 + 0,3i \\ -0,2 + 0,2i \\ -0,2 + 0,2i \\ -0,15 + 0,3i \end{pmatrix}; \quad \kappa_{13} = \begin{pmatrix} -0,25 + 0,3i \\ -0,2 + 0,1i \\ -0,1 + 0,15i \\ -0,2 + 0,1i \end{pmatrix}; \quad \kappa_{23} = \begin{pmatrix} -0,65 + 0,5i \\ -0,65 + 0,5i \\ -0,15 + 0,1i \\ -0,15 + 0,1i \end{pmatrix};$$

$$\kappa_{21} = \begin{pmatrix} -0,15 + 0,2i \\ -0,15 + 0,2i \\ -0,1 + 0,1i \\ -0,1 + 0,1i \end{pmatrix}; \quad \kappa_{31} = \begin{pmatrix} -0,3 + 0,2i \\ -0,25 + 0,15i \\ -0,1 - 0,1i \\ -0,25 + 0,15i \end{pmatrix}; \quad \kappa_{32} = \begin{pmatrix} -0,55 - 0,15i \\ -0,25 + 0,15i \\ -0,25 + 0,15i \\ -0,55 - 0,15i \end{pmatrix}$$

The results of the numerical calculation are indicated by red dots (c). Eigenvalues of the azimuthally non-uniform distributions are marked with crosses, and eigenvalues of the azimuthally uniform amplitude distributions of the lattice (b) are marked with circles.

$$\lambda_o^{sj} = i\tilde{k}_s + \sum_{v=1}^{N-1} \kappa_v^s \cdot (\eta_j)^v \quad (16)$$

Where λ_o^{wj} eigen walue of w-th ring sublattice for $j \neq 0$ oscillations.

Azimuthally homogeneous" solutions are found by assuming: $b_t^s = b^s$ for $s > 0$. Rewriting equations (13) as:

$$\begin{cases} (i\tilde{k}_0 - \lambda)b_0 + N\sum_{s=1}^{M} \kappa_{s0} b^s = 0 \\ (i\tilde{k}_1 - \lambda)b^1 + \kappa_{01}b_0 + (\sum_{n=1,(n\neq t)}^{N} \kappa_{nt}^1) b^1 + \sum_{s=2}^{M} (\sum_{n=1}^{N} \kappa_{nt}^{s1}) b^s = 0 \\ \vdots \\ (i\tilde{k}_M - \lambda)b^M + \kappa_{0M}b_0 + (\sum_{n=1,(n\neq t)}^{N} \kappa_{nt}^M) b^M + \sum_{s=1}^{M-1} (\sum_{n=1}^{N} \kappa_{nt}^{sM}) b^s = 0 \end{cases} \quad (17)$$

As result we obtained a system of M + 1 equations for azimuthally uniform distributions of the amplitudes of a structure consisting of M ring sublattices and one central resonator.

At that obtained number of azimuthally inhomogeneous oscillations is M (N-1), and the total number of oscillations of the system MN +1 resonators is equal: M(N-1) + (M+1) = MN+1.

The coupling matrix in this case takes the form:

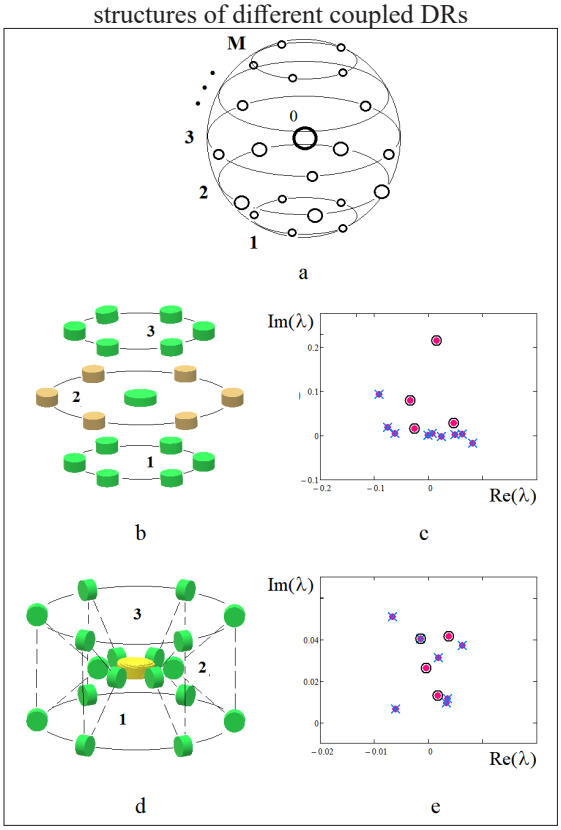
$$K = \begin{pmatrix} i\tilde{k}_0 & N\kappa_{10} & \dots & N\kappa_{M0} \\ \kappa_{01} & i\tilde{k}_1 + \sum_{n=0}^{N-1} \kappa_n^1 & \dots & \sum_{n=0}^{N-1} \kappa_n^{M1} \\ . & . & \dots & . \\ \kappa_{0M} & \sum_{n=0}^{N-1} \kappa_n^{1M} & \dots & i\tilde{k}_M + \sum_{n=0}^{N-1} \kappa_n^M \end{pmatrix} \quad (18)$$

and in a more compact form:

$$K = \begin{pmatrix} i\tilde{k}_0 & N\kappa_{10} & \dots & N\kappa_{M0} \\ \kappa_{01} & \lambda_o^{10} & \dots & \sqrt{N}(\mathbf{K}^{M1}, \mathbf{b}_o^0) \\ . & . & \dots & . \\ \kappa_{0M} & \sqrt{N}(\mathbf{K}^{1M}, \mathbf{b}_o^0) & \dots & \lambda_o^{M0} \end{pmatrix} \quad (19)$$

Fig. 2, c shows the results of comparison of the frequency parameters natural oscillations of three or four resonator ring sublattices with an additional resonator in the center, calculated using the eigenvalues of matrices (14) (crosses), (18-19) (circles) and numerically (dots).

Figure 3: Three dimensional spatial coaxial-multi ring structures of different coupled DRs



Caption: Generalized Three-dimensional coaxial-ring structure of coupled DRs (a). Coaxial-three-ring-spherical structure (b) eigen oscillation parameters (c); coaxial-thoroide structure (d) parameters (e) of the Cylindrical DRs: (N =6; M =3) in the open space, corresponding to the H_{101} oscillations. Frequency: f_o = 8GHz; dielectric permittivity ε_{lr} = 36 ; relative dimensions: $\Delta = L / 2_{ro} = 0, 44$; L - height, 2_{ro} - diameters of the dielectric cylinders; k_0R = 1,75 relative radius of the spherical surface (k0 = ω_0 / c). Eigenvalues corresponding to azimuthally non-uniform distributions are marked with crosses, while those corresponding to azimuthally uniform amplitude distributions of the lattice are marked with circles. The results of the numerical calculation are indicated by red dots.

Eigen oscillations of multi linear-multi ring coaxial lattices of dielectric resonators

We also considered a finite one-dimensional lattice of the DRs, coaxial with M ring sublattices with the same number N of elements are located (Fig. 4, a).

We denoted the coefficients of mutual coupling between the resonators of the linear lattice κ_{nt}^{L}, and the coefficients of mutual coupling between the resonator of the linear lattice and the *t* resonator of the w-th ring sublattices by κ_{st}^{Lw} and vice versa, the coefficients of mutual coupling between the *t* resonator of the w-th ring sublattices and the *s* resonator of the linear lattice by κ_{ts}^{wL} . Then the system of equations for coupled oscillations of a linear-ring structure was represented as:

$$\begin{cases} (i\tilde{k}_0 - \lambda)b_t^0 + \sum\limits_{n=1,(n\neq t)}^{L} \kappa_{nt}^{L}\, b_n^0 + \sum\limits_{s=1}^{M}\sum\limits_{n=1}^{N} \kappa_{nt}^{sL}\, b_n^s = 0 \\ (i\tilde{k}_w - \lambda)b_t^w + \sum\limits_{n=1,}^{L} \kappa_{nt}^{Lw}\, b_n^0 + \\ + \sum\limits_{n=1,(n\neq t)}^{N} \kappa_{nt}^{w}\, b_n^w + \sum\limits_{s=1,(s\neq w)}^{M-1}\sum\limits_{n=1}^{N} \kappa_{nt}^{sw}\, b_n^s = 0 \end{cases} \quad (20)$$

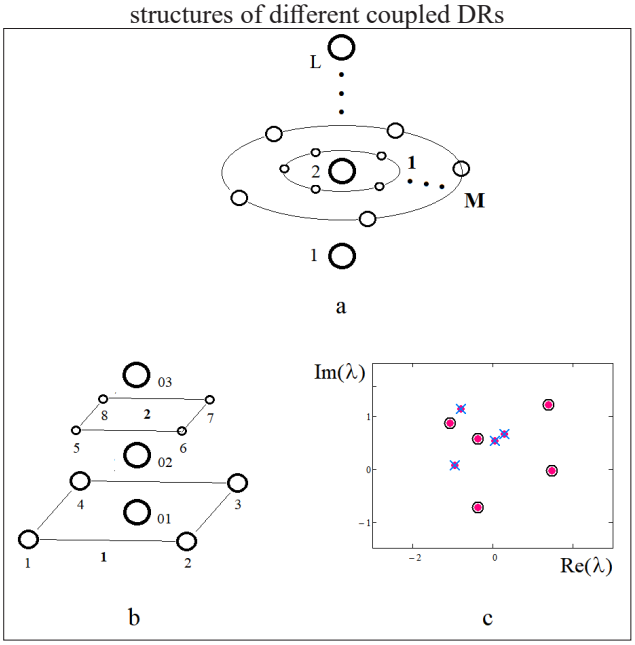
In the case where the coupling coefficients between the fixed resonator of the linear lattice and each of the ring sublattices are the same:

$$\kappa_{nt}^{sL} = \kappa_{t}^{sL}\ ;\ \kappa_{nt}^{Lw} = \kappa_{n}^{Lw}$$

equations (20) take the form:

$$\begin{cases} (i\tilde{k}_0 - \lambda)b_t^0 + \sum\limits_{n=1,(n\neq t)}^{L} \kappa_{nt}^{L}\, b_n^0 + \sum\limits_{s=1}^{M} \kappa_{t}^{sL} \left(\sum\limits_{n=1}^{N} b_n^s\right) = 0 \\ (i\tilde{k}_w - \lambda)b_t^w + \sum\limits_{n=1,}^{L} \kappa_{n}^{Lw}\, b_n^0 + \\ \sum\limits_{n=1,(n\neq t)}^{N} \kappa_{nt}^{w}\, b_n^w + \sum\limits_{s=1,(s\neq w)}^{M-1}\sum\limits_{n=1}^{N} \kappa_{nt}^{sw}\, b_n^s = 0 \end{cases} \quad (21)$$

Figure 4: Three dimensional multi linear-multi ring coaxial structures of different coupled DRs



Caption: Generalized Three-dimensional multi linear-multi ring coaxial structures of coupled different DRs (a). Three linear-two ring structure of coupled DRs (b): N = 4; M = 2; L = 3. The coupling coefficients are arbitrary complex numbers:

$\tilde{k}_0 = 0,5$; $\tilde{k}_1 = 0,25$; $\tilde{k}_2 = 0,75$; $\kappa_{12}^{L} = -0,8 + 0,3i$; $\kappa_{13}^{L} = 0,3 + 0,2i$; $\kappa_{23}^{L} = -0,4 + 0,25i$

$\kappa_1^{L1} = 0,5 + 0,25i$; $\kappa_2^{L1} = 0,3 + 0,2i$; $\kappa_3^{L1} = -0,2 + 0,25i$

$\kappa_1^{L2} = 0,4 + 0,2i$; $\kappa_2^{L2} = 0,3 - 0,2i$; $\kappa_3^{L2} = -0,2 + 0,25i$;

$\kappa_1^{1L} = 0,3 + 0,2i$; $\kappa_2^{1L} = 0,2 + 0,2i$; $\kappa_3^{1L} = 0,2 + 0,2i$;

$\kappa_1^{2L} = 0,2 + 0,2i$; $\kappa_2^{2L} = 0,3 - 0,25i$; $\kappa_3^{2L} = -0,2 + 0,15i$;

$$\kappa_1 = \begin{pmatrix} 0,4-0,5i \\ -0,2-0,1i \\ 0,4-0,5i \end{pmatrix};\ \kappa_2 = \begin{pmatrix} 0,3+0,3i \\ -0,15-0,1i \\ 0,3+0,3i \end{pmatrix};\ \kappa_{12} = \begin{pmatrix} 0,5+0,25i \\ 0,15+0,25i \\ 0,2+0,2i \\ 0,15+0,25i \end{pmatrix};$$

$$\kappa_{21} = \begin{pmatrix} 0,35+0,15i \\ 0,25-0,15i \\ 0,1+0,1i \\ 0,25-0,15i \end{pmatrix}$$

Eigenvalues of the azimuthally non-uniform amplitude distributions are crosses; eigenvalues of the to azimuthally uniform amplitude distributions of the structure (b) are circles.

The results of the numerical calculation are red dots.

We are looked for a solution (21) for ring sublattice amplitude resonators in the form:

$$b_t^v = a^v b_o^t \qquad (22)$$

where the amplitudes a^v do not depend on the DR number *t*.

We substituted (22) into (21) and take into account (16):

$$\begin{cases} (i\tilde{k}_0 - \lambda)b_t^0 + \sum_{n=1,(n\neq t)}^{L} \kappa_{nt}^{L}\, b_n^0 + \sum_{s=1}^{M} \kappa_t^{sL} a^s (\sum_{n=1}^{N} b_o^n) = 0 \\ (\lambda_o^w - \lambda)a^w b_o^t + \sum_{n=1,}^{L} \kappa_n^{Lw}\, b_n^0 + \sum_{s=1,(s\neq w)}^{M-1} (\sum_{n=1}^{N} \kappa_{nt}^{sw} b_o^n) a^s = 0 \end{cases} \qquad (23)$$

In the case of an azimuthally non-uniform amplitude distributions in ring sublattices:

$$\sum_{n=1}^{N} b_o^n = 0 \quad \text{for } j \neq 0 \qquad (24)$$

therefore from (23)

$$\begin{cases} (i\tilde{k}_0 - \lambda)b_t^0 + \sum_{n=1,(n\neq t)}^{L} \kappa_{nt}^{L}\, b_n^0 = 0 \\ \sum_{n=1,}^{L} \kappa_n^{Lw}\, b_n^0 + (\lambda_o^w - \lambda)a^w b_o^t + \sum_{s=1,(s\neq w)}^{M-1} (\sum_{n=1}^{N} \kappa_{nt}^{sw} b_o^n) a^s = 0 \end{cases} \qquad (25)$$

The matrix of the system is reduced to a block matrix, and its determinant, takes the form of a product of two quantities, one of which corresponds to the matrix of the linear sublattice, and the other to the determinant of the matrix, determining the coupled oscillations of the ring sublattices.

The solution for an azimuthally uniform distributions of resonator amplitudes in the ring sublattices takes the form:

$$\sum_{n=1}^{N} b_o^n = N \cdot b_o^n$$

:

$$\begin{cases} (i\tilde{k}_0 - \lambda)b_t^0 + \sum_{n=1,(n\neq t)}^{L} \kappa_{nt}^{L}\, b_n^0 + N \cdot b_o^n \sum_{s=1}^{M} \kappa_t^{sL} a^s = 0 \\ (\lambda_o^w - \lambda)a^w b_o^n + \sum_{n=1,}^{L} \kappa_n^{Lw}\, b_n^0 + b_o^n \sum_{s=1,(s\neq w)}^{M-1} (\sum_{n=1}^{N} \kappa_{nt}^{sw}) a^s = 0 \end{cases}$$

Since all the amplitudes of the resonators inside the ring sublattices are the same, we can make a substitution of variables: $\beta_t^0 = b_t^0 / b_o^n$. Moreover, for coaxial ring sublattices, the sum $\sum_{n=1}^{N} \kappa_{nt}^{sw} = q^{sw}$ does not depend on the resonator number *t*.

Taking into account the comments made, we obtained a system of L + M equations.

$$\begin{cases} (i\tilde{k}_0 - \lambda)\beta_t^0 + \sum_{n=1,(n\neq t)}^{L} \kappa_{nt}^{L}\, \beta_n^0 + N \cdot \sum_{s=1}^{M} \kappa_t^{sL} a^s = 0 \\ \sum_{n=1,}^{L} \kappa_n^{Lw}\, \beta_n^0 + (\lambda_o^w - \lambda)a^w + \sum_{s=1,(s\neq w)}^{M-1} q^{sw} a^s = 0 \end{cases} \qquad (26)$$

with respect to unknown amplitudes of resonators of a linear sublattice $\{b_t^0 / b_n^0\}$(t = 1,2,..., **L**); and amplitude multipliers (22) of ring sublattices of resonators $\{a^w\}$ (w = 1,2,..., **M**).

Thus, from the system of equations (25) we found solutions: $b_t^0 = 0$ for the amplitudes of the resonators of the linear sublattice (t = 1,2,..., **L**) and M x (N-1) azimuthally inhomogeneous distributions of amplitudes of coupled ring sublattices (Trubin, 2025).

Total number of natural oscillations of the structure: **(L + M) + M x (N-1) → L + M x N.**

Fig. 4, c shows the result of calculating the eigenvalues λ for two ring lattices consisting of 4 DRs with one-dimensional three DR sublattice, obtained numerically (dots) and found from (23) – cicles; [24] – crosses. The obtained data show good agreement between the analytical and numerical solutions.

Discussion

The obtained results allows us to formulate conditions whose fulfillment leads to simplification of equation systems describing the coupled oscillations of new class complex linear-ring structures of dielectric resonators. The specified condition is the equality of the coupling coefficients between each of the linear sublattice resonators and the resonators of the ring sublattices. In this case, the complex structure experiences oscillations that are completely identical to the oscillations of the ring sublattices, characterized by an azimuthally nonuniform distribution of the resonator amplitudes. The linear sublattices participate only in oscillations with an azimuthally uniform amplitude distributions.

Conclusion

- A theory of the eigen frequencies and eigen amplitudes of coupled oscillations of complex DR linear-ring structures using perturbation theory has been developed.
- From general perturbation theory, that are suitable for obtaining analytical solutions, the systems of equations are derived in a simpler form.
- It has been shown that if the coupling coefficients between the resonators of linear and ring lattices are identical, the natural oscillations of linear-ring structures decompose into two types. Oscillations with a nonuniform azimuthal distribution of the resonator amplitudes of the ring sublattices are characterized by zero resonator amplitudes of the linear sublattice. In the case of an uniform azimuthal distributions of the resonator amplitudes, the linear and ring sublattices participate in coupled oscillations.
- The effectiveness of the proposed method is demonstrated by presented examples of solutions and their comparison with data obtained from numerical calculations.

- The developed theory serves as the basis for calculating the parameters of coupled oscillations of complex multi-element DR lattices and is the basis for optimizing the parameters of a wide class of different microwave, infrared and optical devices for various communication systems build based on different DRs.

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